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IN–PLANE AND OUT–OF–PLANE STABILITY OF CURVED BEAMS – AN OVERVIEW

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Abstract: The current review paper summarizes a wide variety of research articles that focus on different stability issues of curved beams. It delves into various theories and solutions that got in the spotlight on curved beam stability during the last few decades. This brief review article brings together the major findings of in–plane and out–plane curved beam stability from several scientific perspectives, including analytical, numerical and experimental investigations of either homogenous or nonhomogeneous materials under multiple load cases and different support conditions.

Keywords: review, curved beam, arch, stability, buckling

INTRODUCTION

Euler, the pioneer of beam buckling, presented his famous formula for the critical (buckling) load of straight columns under compression in 1757 [1]. Since then, his finding has ignited interest in this field and a large number of new models have emerged in the literature. Because of the initial curvature, curved beams act in a different way under mechanical loads in comparison with straight beams which led to garnering considerable interest from researchers. The usage of curved beams is continuously growing in popularity for a variety of reasons, including their beneficial mechanical behaviour, particularly when the dominant load results in compression, and the appreciated aesthetics of curved elements in contemporary architecture is also a factor to be considered. In the aerospace, civil, and marine engineering sectors, the curved beams are a widely used structural element [2,3]. Many infrastructure systems, such as long–span roofs, utilize the arch as a structural shape and as a result, in engineering design, a thorough knowledge of its stability behaviour under various loading conditions is crucial.

Many researchers have studied numerous kinds of curved beam behaviour to understand the complexity and give engineers practical knowledge and adequate information on the stability. To mention some classical findings, Simitses [4] and Timoshenko and Gere [5] studied the classical theory extensively to predict elastic buckling loads and they came up with an approximation for the classical buckling load for sinusoidal shallow arches under evenly distributed load. Many researchers expanded the above mechanical approach, resulting in closed–form solutions. There were several attempts to deal with the nonlinear arch stability problem using the finite element method under a variety of assumptions [6,7]. In order to avoid arch buckling, it is essential to be able to accurately predict the buckling load required for the resistance design [8].

The current article aims to give an insight to the most popular research findings about the stability of curved beams. It is divided into two major parts, as the results are gathered and distinguished as in–plane and out –of plane stability issues.

IN–PLANE STABILITY OF CURVED BEAMS

The examination of a curved beam's in–plane stability is a classical topic in applied mechanics. Curved beams may buckle in a snap–through mode or bifurcation mode. Study [9] focused on pin–ended and fixed uniform circular arches with any arbitrary symmetric cross–section subjected to a radial force distributed uniformly around the arch axis. Both non–linear equilibrium equations and buckling equations for shallow and non–shallow arches were established using a variational principle. Both analytical and approximate solutions were obtained and proposed, verified by finite element computations. Additionally, the characteristics that distinguish between shallow and non–shallow arches were also mentioned. It was discovered that since the classical linear buckling theory does not take into consideration the pre–buckling deformations, it couldn’t be utilized to predict the in–plane buckling of shallow arches accurately. The buckling load of non–shallow arches might, however, be predicted by the linear theory since their pre–buckling deformations are not that relevant. The stability of a uniform half–sine shallow arch was examined under static loading in a thermal environment in study [10]. The arch had pinned supports and the material was linearly elastic, isotropic. The kinematical model was based on a modified Euler–Bernoulli theory, assuming large transverse displacements. Furthermore, the axial force was assumed to be constant throughout the arch. The effect of three mechanical loading types (concentrated, uniform distributed and asymmetrical distributed) were examined by tracking the equilibrium paths of the arches. The findings revealed that shallow arches behaved similarly under concentrated loading at the midpoint and uniformly distributed loading. Rubin [11] used the Cosserat theory, a special continuum theory with its own balance rules (conservation of mass, balances of linear and angular momentum, and balance of director momentum), to estimate the buckling loads and deformed shapes of elastic clamped circular arches. Those predictions were in a very good agreement with the experiments across the entire range of arch geometries tested by Gjelsvik and Bodner [12]. In the experiments, the arches were made of 2024–T4 aluminum plate that was sliced into strips of constant width and then formed into circular arches of various radius under displacement control. The arch was split into N equal elements so that the experiment might be modeled using the theory of a Cosserat point. It was demonstrated, in particular, that the theory of a Cosserat point gave predictions of the buckling loads and buckled shapes by utilizing a modified thickness that was within the range of manufacturing tolerances of the plates. The results showed that the buckling force was sensitive to the thickness of the plate used to build the arches.

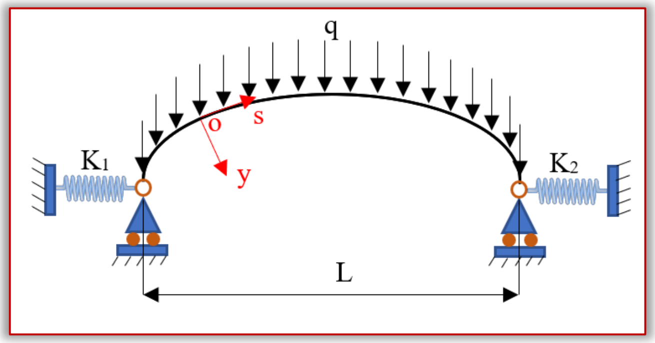

Figure 1. A shallow parabolic arch with horizontal spring supports [13]

In paper [13], the authors developed an analytical model using the virtual work principle for the in–plane elastic stability of a shallow parabolic arch supported by horizontal springs. The arch was subjected to a uniformly distributed load as depicted in Figure 1. They specifically looked at the impact of the stiffness of the horizontal springs on the buckling behavior. It was assumed that (a) the vertical displacements at the ends were completely prevented by roller supports, but rotations at the ends might happen freely, (b) the stiffness of both horizontal springs was equal with each other so the arch flexes symmetrically before buckling, and (c) the arch material was linear elastic. The stiffness of the horizontal springs were discovered to have a significant impact on the buckling load.

Cai and Feng [14] examined the nonlinear in–plane stability of parabolic shallow arches with elastic supports under a central concentrated force. The nonlinear equilibrium equation and the buckled equilibrium equation were established using the virtual work principle. It was assumed that k is the stiffness of the rotational elastic spring which is defined by $k=(\alpha+\eta^2\beta)\ EI_z/L$. Here η denoted the stiffness change of the structural supports with respect to the axial force; α and β are the initial stiffness coefficient and the stiffening rate, while I_zE is the bending stiffness and L is the initial arch length. The critical loads were determined for both symmetric snap–through buckling and antisymmetric bifurcation buckling. The critical load increased when either the initial stiffness of the elastic supports or its stiffening rate increased. It was found that the rotational stiffness has a more severe impact on antisymmetric buckling than on symmetric buckling. With pinned ends, the critical load for any stiffening rate is lower, while with clamped ends, it is greater.

Researches on the stability of arches made of functionally graded materials (FGM) can also be found in the open literature. A non–linear stability study of FGM (constituted of metal and ceramic phases) circular shallow arches under central concentrated force was presented in [15]. The boundary conditions of simple support and clamped–clamped end was assumed, and material characteristics changed according to a power–law distribution across the arch thickness. To approximate the displacement field over the arch, the conventional single layer theory was applied as per the Donnell shallow shell theory together with von–Karman type geometrical nonlinearities. Small strains and moderately large rotations were assumed. All the stretching, coupled stretching–bending and bending stiffnesses were accounted in the model. The governing static equilibrium equation yielded the classical finding that the axial force is constant along the centerline. The impact of material dispersion, geometrical parameters, and boundary conditions on the stability of shallow arches subjected to a central concentrated force was investigated. A change in the power–law index in the Voigt rule of mixture could significantly affect the stability behavior of the arches. Furthermore, the results showed that the elasticity modulus ratio of the constituents also had an impact on the findings. Under a central point force, the authors of paper [16] examined the instability of functionally graded multilayer composite shallow arches reinforced with a low amount of graphene platelets (FG–GPLRC in short). The nonlinear equilibrium equations for the FG–GPLRC arch, fixed or pinned at both ends, were established using the virtual work concept and then solved analytically. GPL nanofillers were discovered to have a significant reinforcing impact on the buckling and post–buckling performances of nanocomposite shallow arches. The buckling and post–buckling behaviors were analyzed in relation to the distribution pattern, weight percentage, and size of GPL nanofillers as well as the geometrical parameters of the arch. No matter how GPLs were distributed, all FG–GPLRC arches loaded by a central point force had a considerably greater limit point buckling load than a pure epoxy arch, demonstrating the strong reinforcing impact of GPL nanofillers. Limit point buckling load and bifurcation load increased as GPL weight fraction increased and pattern X had a greater reinforcing effect among the three GPL distribution patterns evaluated (U, X, and O).

Guo et al. [17] examined the nonlinear behaviour of fixed parabolic shallow arches under a vertical uniform load to determine the in–plane buckling behaviour. The non–linear equilibrium and buckling equations were developed using the virtual work principle. For the non–linear in–plane symmetric snap–through and antisymmetric bifurcation buckling loads, analytical solutions were found. The finite element package ANSYS was used in the numerical analysis where I– and circular sections were considered. Investigations were done on the connection between the slenderness and dimensionless buckling loads. It was demonstrated that the results of the model agreed well to the numerical outcomes. The authors suggested to use cables in order to enhance the in–plane performance of the parabolic shallow arches and it was concluded from the finite element results that although the cable's impact on the symmetric buckling was minor, it was more noticeable as the modified slenderness of the arch increased.

The in–plane elastic static stability of circular beams with cross–sectional inhomogeneity when exposed to a vertical load at the crown point, was investigated by Kiss [18]. He developed a novel non–linear model for shallow curved beams based on the principle of virtual work to determine the critical loads both for symmetric snap–through and antisymmetric bifurcation buckling for pinned–pinned, fixed–fixed and rotationally restrained supports. The buckling ranges and its ends points, which were dependent on the supports and the geometry parameter, were investigated. The results were compared with those in the literature and with the use of commercial FE software to confirm that the new model was more accurate to predict the critical loads for the above–mentioned beams. The coupling effects of intrinsic geometric non–linearity and extra complexity, such as temperature variations have gotten scientific attention. The in–plane stability of rotationally restrained shallow arches subjected to temperature variations and a vertical uniform mechanical load was investigated by Cai et al [19]. The nonlinear equilibrium and buckling equations were established using the virtual work principle, and analytical solution for the nonlinear in–plane symmetric and asymmetric bifurcation critical loads were found. Temperature variations had a considerable impact on the critical loads for both the symmetric snap–through and asymmetric bifurcation modes. For arches under uniform temperature field and mechanical load, the limit points and bifurcation points appeared at higher load levels than for only externally loaded members. An increase in rotational stiffness of supports resulted an increase in the impact of the uniform temperature field, but it resulted a decrease in the effects of the temperature gradient. Hu and Huang [20] proposed an analytical method for the in–plane nonlinear elastic buckling and post–buckling of pin–ended parabolic multi–span continuous arches. On the basis of the virtual work principle and the non–linear strain expression, the in–plane nonlinear equilibrium differential equations of were developed. The deformed buckled symmetric or asymmetric shape of pin–ended parabolic multi–span continuous arches varies notably from single arches – see Figure 2. The results of the buckling load were in good agreement with the finite element results. Their research can be extended by taking into account the lateral torsional deformations and the inhomogeneous material to study the out of– plane stability of multi –span arches.

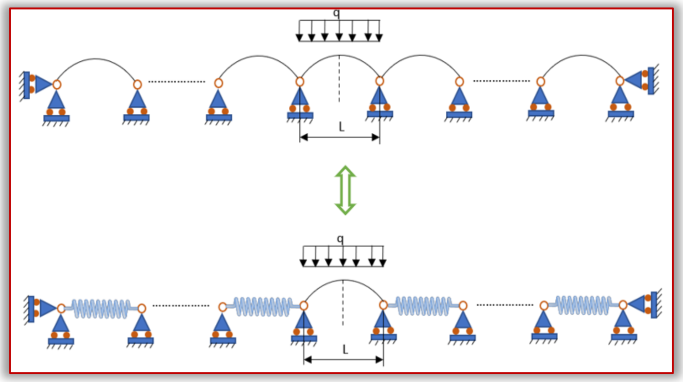

Figure 2. Nonlinearity effect of multi–span continuous arch [20]

A theoretical investigation of the linear and nonlinear elastic in–plane behavior of pinned–fixed circular arches subjected to a radial load distributed uniformly around the arch axis was reported in paper [21]. It was presumed that the arch's deformations adhere to the Euler–Bernoulli hypothesis and the cross–section's height was much smaller than the circular arch's original radius. It was discovered that a pinned–fixed arch could buckle in a limit point mode but not in a bifurcation mode when applied to a uniform radial load. That was quite different to pinned–pinned or fixed–fixed arches subjected to a uniform radial load, which might buckle in a bifurcation and limit point mode. Analytical solutions accurately described the shallow pinned–fixed arches' nonlinear buckling and post–buckling behavior, according to comparisons with finite element findings. In paper [22], the nonlinear elastic in–plane dynamic buckling of a fixed shallow circular arch was investigated when it was subjected to an arbitrarily located step radial point load. Analytical solutions for the nonlinear dynamic buckling load of the arch were derived using the principle of the conservation of energy. It was discovered that the load position's asymmetric effects on the dynamic buckling load were considerable. It turned out that as the load application point moved further away from the crown, the dynamic buckling load initially dropped and subsequently increased. Additionally, the effects of the modified slenderness on dynamic buckling were examined, and it was shown that as the modified slenderness increased, the dynamic buckling load also increased.

Novel functionally graded porous (FGP) materials have been suggested and developed by many researchers to combine the benefits of functionally graded materials and porous materials in the production of lightweight structures [23]. Under a time–varying uniform radial pressure, the dynamic instability behavior of FGP pinned arch reinforced with uniformly distributed graphene platelets (GPLs) was investigated in the study [24] using Euler–Bernoulli theory, Hamilton's principle, the Galerkin technique, and Bolotin's approach. The symmetric, asymmetric, and uniform porosity distributions of GPLs reinforced arches were examined. The effect of GPL weight fraction and dimensions, porosity distribution, pore size, and arch geometry on the dynamic stability characteristics of the arch were studied. The arch with a symmetric porosity distribution was shown to be more stable than the other two porosity distributions. A bigger unstable region was associated with a higher porosity coefficient. The inclusion of minor numbers of GPLs improved the arch's stability performance considerably, according to numerical findings. The impacts of GPL shape and size were shown to be far less important.

OUT–OF–PLANE STABILITY OF CURVED BEAMS

The usage of single freestanding arches has risen in recent decades. Because these arches lack lateral bracing, they are susceptible to out–of–plane buckling movements and failure. Out–of–plane buckling is one of the buckling modes that can take place when an arch is subjected to a combination of bending and compression [25]. A number of analytical and numerical studies on elastic and inelastic out–of– plane buckling has been published in the literature. In paper [26], the authors investigated the out–of–plane elastic and inelastic stability of concrete–filled steel tubular (CFST) circular arches as shown in Figure 3. The effect of uniformly distributed radial loading or central concentrated loads and elevated temperature fields was investigated. An elastic pre–buckling and out–of–plane buckling analysis was carried out using energy techniques, providing numerical systems to determine the buckling loads for pinned /fixed–ended shallow arches. It was discovered that, in contrast to steel arches, the elastic buckling loads of CFST arches were sensitive to temperature. For the uniformly distributed radial loading case, the effects of thermal loads increased with arch slenderness. However, this thermal load on the buckling strength was unaffected by the arch slenderness in the case of central concentrated loads. Using FE analysis, it was determined how temperature changes affected the out–of–plane buckling loads of CFST arches and the numerical system developed for the calculation of elastic buckling loads was used to validate the FE model. The author can extend his research based on this paper's findings to study the out–plane stability of CFST arches for other support circumstances like elastic supports and take into account that the temperature field varies through the length of the arch.

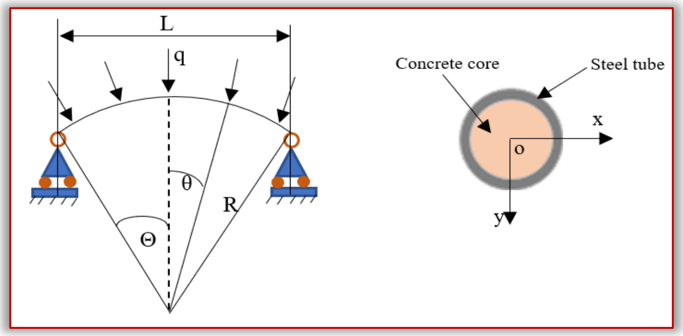

Figure 3. Geometrical and loading conditions for a pin–ended shallow CFST arch [26]

Study [27] examined a fixed CFST parabolic arch under uniformly distributed loads. Using a test–verified FE approach, authors analyzed the out–of–plane buckling behavior of CFST arches with various rise–to–span ratios while taking material and geometric non–linearities into account. It was pointed out that the axial force, out–of–plane bending moment, and torque all increased when the rise–to–span ratio increased.

The stability of CFST arches for long–span arch bridges may be impacted by the time–dependent behavior of the core concrete. Using the finite element technique, Geng et al. [28] looked at how pre–buckling deformations driven by time effects impact the out–of–plane stability of single parabolic arches with fixed ends and uniformly distributed loads were applied throughout the span. It was assumed that plane sections would stay plane and that there would be no slipping or separation of the steel tube from the concrete core. Timoshenko beam elements were used to model the concrete core and the steel tubes in ABAQUS. It turned out that the pre–buckling deformations induced by time effects can reduce the arch's ultimate capacity by up to 18%. On the basis of the findings from the finite element analysis, designing equations were suggested to estimate the ultimate loads while taking time effects into consideration. In study [29], an experimental investigation was reported on the inelastic buckling behavior of fixed circular steel I–cross–section arches under symmetric three–point loading and non–symmetric two–point loading as depicted in Figure 4.

The out–of–plane inelastic buckling strength of fixed steel arches was found to be significantly influenced by the geometric imperfections, the out–of–plane elastic buckling modes and in–plane loading patterns, according to the experimental results and commercial FE investigations. It was discovered that the FE findings of the buckling strength, with relative errors of less than 8%, correlate quite well with the test results. Fixed arches under non–symmetric two–point load had lower out–of–plane buckling strengths than that under symmetric three–point loading.

A fixed steel arch may fail under in–plane stress in an out–of–plane inelastic buckling if it lacks sufficient lateral bracings. In this case, their research can be extended to examine the impact of bracing factors, such as bracing stiffness, brace types, the number and longitudinal position of bracing points, on the arches' out–of–plane stability.

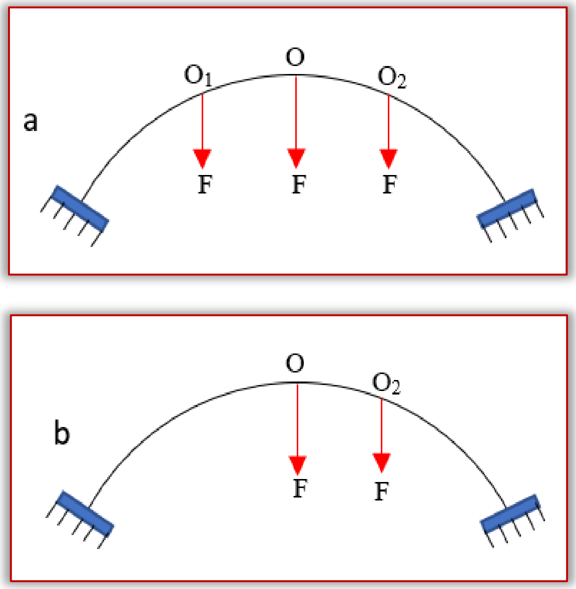

Figure 4. (a) Symmetric three–point loading;
(b) non–symmetric two–point loading [29]

Pi and Trahair [30] presented a nonlinear inelastic approach for studying circular arches' out–of–plane inelastic flexural–torsional behavior. Subtended angle, in–plane curvature, initial crookedness, and material inelasticity were all taken into consideration to examine the strength of steel arches in uniform compression and in uniform bending.

By including material inelasticity into a nonlinear elastic finite–element model for arches [31], a nonlinear finite–element model for the three–dimensional large deformation analysis of circular arches was created. It was discovered that the in–plane curvature and subtended angle had a substantial impact on the inelastic buckling and strength of arches. It was shown that when the subtended angle and in–plane curvature increased, the strength of the arches also dropped. When the subtended angles or in–plane curvatures are unchanged, the reduced slenderness of arches caused a decrease in their out–of–plane strengths in uniform compression and in uniform bending. Both in uniform compression and bending, the effects of initial crookedness on the out–of–plane strengths were considerable for arches.

The AS4100 design guidelines [32] for steel beams do not immediately apply to the design of steel arches under uniform compression and uniform bending since they do not take the effects of in–plane curvature into account. So the specified equations to their model could cover this gap. In many circumstances, circular arches that are loaded in–plane are constrained with discrete lateral bracings to increase their out–of–plane stability. If the effective length of the arch segment between two consecutive bracings is sufficiently great, the arch section may bend out of plane. Adjacent arch segments can act as elastic end restraints for the ends of the arch segment during the out–of–plane buckling of an arch segment.

Out–of–plane elastic buckling of circular arch segments with elastic end restraints that were subjected to a uniform radial force, as shown in Figure 5, was the subject of paper [33]. Authors developed an approximate solution for the out–of–plane elastic buckling forces by utilizing the energy method in combination with the Rayleigh–Ritz method. It was assumed that the cross sections were doubly symmetrical and the shear deformation of the cross–section was ignored. The approximate solutions were compared to the FE findings and were found to be accurate enough. It was found that out–of–plane buckling loads increased significantly with an increase in the rotational restraint's coefficient.

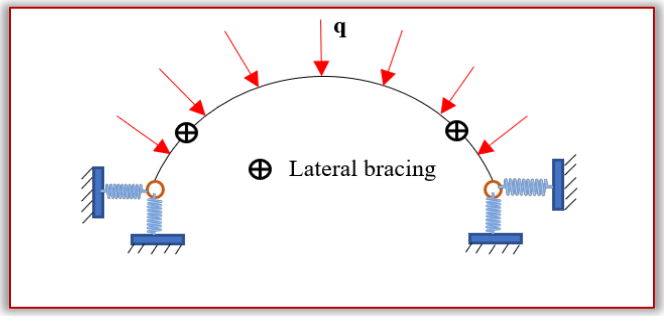

Figure 5. Circular arch with discrete lateral bracings [33]

CONCLUSIONS

This study overviewed certain analytical, numerical and experimental solutions from the open literature of recent decades in the in– plane/out –of– plane stability issues of curved beams. Lot of researches on the stability problems of curved beams have been investigated, each with its unique set of boundary conditions and methods. As shown in the reviewed articles, there is still a lot of interest in the stability of arches and their applications in real–world problems. Some of the proposed methodologies and equations in the reviewed papers can be extended to investigate the dynamic in– or out–of–

plane stability problem of various arches with different cross–sectional shapes, with different profiles (catenary, parabola) under different types of instantaneous loadings.

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