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# PROBABILITY-BASED DESIGN OF A SOLID TIMBER COLUMN SUBJECTED AXIAL COMPRESSION AND BENDING

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**Abstract:** In this paper, the results of the probability-based design of a solid timber column of softwood specie and Strength Class C24 at predefined safety levels under axial compression and bending in accordance with the design rules of EC-5 (1995) are discussed. Different values of coefficient of variation (10%, 15%, and 20%) of the column strength in compression and bending were considered in order to obtain adequate cross sectional dimensions at predefined safety levels. The First Order Reliability method was invoked to check the safety of the designed section using a MATLAB program. It was found that economical column cross sectional dimensions were achieved at lower target safety index. It was also found that the results obtained from the failure of the column due to axial compression were more economical in terms of material consumption compared to the results obtained due to column failure in bending. However, the results obtained all satisfied the design criteria in compression and bending.

**Keywords:** Probability based design, solid timber column, predefined safety levels, first order reliability method, design criteria, EC-5

## INTRODUCTION

The major objective of structural design is the fulfillment of certain performance criteria that relate to safety and serviceability of the structure or structural element (Ogork and Nakore, 2017; Abubakar, 2006; Ranganathan, 1990). Structural problems are non-deterministic in nature (El-Reedy, 2013; Abejide, 2012). In consequence, the use of partial safety factors cannot guarantee the required safety levels as it fails to explicitly consider the probability of failure associated with some performance criteria of the structure (Afolayan and Abubakar, 2003). The violation of ultimate and serviceability limit states of engineered structures may lead to loss of lives and damage of properties worth millions of naira (Sule and Benu, 2012; Sule and Benu, 2019). Hence there is a need to accurately determine the limit state to enhance efficient design. However, the attainment of a limit state is always difficult due to uncertainties inherent in the resistance and load parameters. According to Melchers (1999), the failure of structures may not be related directly to the predicted normal loading or strength. It may result from human error, negligence, poor workmanship or neglected loading. The variability of material properties, fluctuation of loads on structures and uncertainties of the design models cause the performance of the structure to fall below expectation.

A probabilistic approach always provides a rational way of dealing with such uncertainties that are inherent in structures by using statistical approach. A probabilistic design aims at finding the optimal solution that satisfies the prescribed performance criteria (Afolayan, 2002; Afolayan and Abubakar, 2003; Salisu et. al, 2009; Afolayan, 2005). In this paper, a probability-based design of a pin-ended solid square timber column of square cross section under axial compression and bending is carried out using EC-5 design rules.

The variability of the strength of column in axial compression and bending at varying values of load ratio were considered in the probabilistic design of the column cross sectional dimensions. The safety of the designed cross sections was checked at predefined safety level using First Order Reliability procedure coded in a MATLAB language.

## PERFORMANCE CRITERIA

The performance criteria are derived in accordance with the EC-5 design rules for timber structures. A pin-ended solid square timber column under axial compression and bending (Figure 1) is considered in this study.

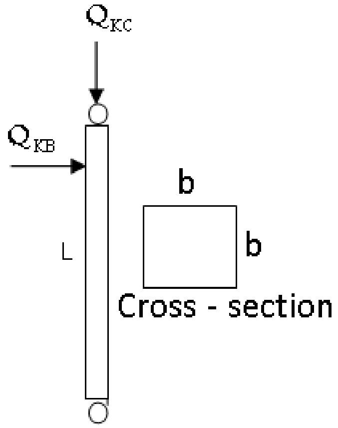

Figure 1. A pin-ended square timber column under axial compression and bending

### — Compression criterion

The design compressive stress in column parallel to grain is given by:

$$\sigma_{c,d} = \frac{Q_C}{A} = \frac{Q_C}{b^2} \tag{1}$$

where $\sigma_{c,d}$ = design stress, $A$ = cross- sectional area, $b$ = cross sectional dimension of the column, $Q_C$ = compressive load on column

The factored compressive load on column is given by:

$$Q_C = Q_{KC}\left(1.4\alpha_C + 1.6\right) \tag{2}$$

Therefore, the compressive strength parallel to the grain is given by:

$$f_{c,d}=\frac{K_{mod}\, f_{c,k}}{\gamma_m} \tag{3}$$

where $K_{mod}$ = modification factor for duration of loading and moisture content, $Q_1$ = short term axial load, $\gamma_m$ = partial safety factor for the material property based on EC-5, $f_{c,k}$ = characteristic value of the compressive strength based on timber strength class, $\alpha_C$ = load ratio under axial compression

Applying equations (1), (3) and (4), the limit state function in axial compression is given by:

$$g(x)=\frac{K_{mod}\, f_{c,k}}{\gamma_m}-\frac{Q_K(1.35\alpha+1.5)}{b^2} \tag{4}$$

— Bending criterion

The applied bending stress parallel to grain is given by:

$$\sigma_{m,d}=\frac{M}{Z} \tag{5}$$

The induced bending moment on beam under uniform loading is given by:

$$M=\frac{Q_L L^2}{8} \tag{6}$$

From applied structural mechanics, the section modulus of a solid square section is given by:

$$Z=\frac{b^3}{6} \tag{7}$$

Applying equations (5), (6) and (7), the load induced bending stress parallel to grain is given by:

$$\sigma_{m,d}=\frac{0.75 Q_L L^2}{b^3} \tag{8}$$

where $Q_L$ = short term lateral load that causes bending, $b$ = cross sectional dimension of column

The factored applied lateral load is given by:

$$Q_L = Q_{KB}(1.4\alpha_B+1.6) \tag{9}$$

According to EC-5, the design bending strength parallel to grain is given by:

$$f_{m,d}=\frac{K_{mod}\, f_{m,k}}{\gamma_m} \tag{10}$$

where $f_{m,k}$ = characteristic value of the bending strength

Applying equations (8), (9) and (10), the limit state function in bending is given by:

$$g(x)=\frac{K_{mod}\, f_{m,k}}{\gamma_m}-\frac{0.75 Q_{KB}(1.4\alpha_B+1.6)L^2}{b^3} \tag{11}$$

## MATERIALS AND METHOD

The First order reliability method is used to obtain the design points on the failure surface. Let the failure surface in x-space be given by:

$$g(x) = g(x_1, x_2, \ldots, x_n) = 0 \tag{12}$$

The vector of the random variables in x-space is given by:

$$x = [x_1, x_2, \ldots, x_n]^T \tag{13}$$

The normalized random variables $y_1, y_2, \ldots, y_n$ are introduced using an appropriate one to one linear mapping in the form of $x = L(y)$ such that $y = L^{-1}(x)$. The corresponding design points in y-space are then defined by the transformation:

$$x = L(y), \quad y = L^{-1}(x) \tag{14}$$

Consequently, equation (12) maps equation (14) into:

$$h(y_1, y_2, \ldots, y_n) = 0 \tag{15}$$

The function h is defined by:

$$h(y) = g[L(y)] \tag{16}$$

Equation (16) is the failure function in normalized coordinate. The minimum distance between the origin and the failure surface in normalized coordinate is the reliability index, $\beta$.

The reliability index, $\beta$ is given by:

$$\beta = \min\left\langle \sqrt{\sum y_1^{\,2} + y_2^{\,2} + \ldots + y_n^{\,2}} \,\middle|\, h(y_1, y_2, \ldots, y_n) \right\rangle = 0 \tag{17}$$

The values of the design variables that minimize the distance from the origin to the failure surface subject to $h(y_1, y_2, \ldots, y_n) = 0$ are obtained by iteration scheme.

The design is adequate when:

$$\beta = \min\left\langle \sqrt{\sum y_1^{\,2} + y_2^{\,2} + \ldots + y_n^{\,2}} \,\middle|\, h(y_1, y_2, \ldots, y_n) \right\rangle \approx \beta^T \tag{18}$$

The flowchart that shows the design procedure and statistics of the basic variables are shown in Figure 1 and Table 1 respectively.

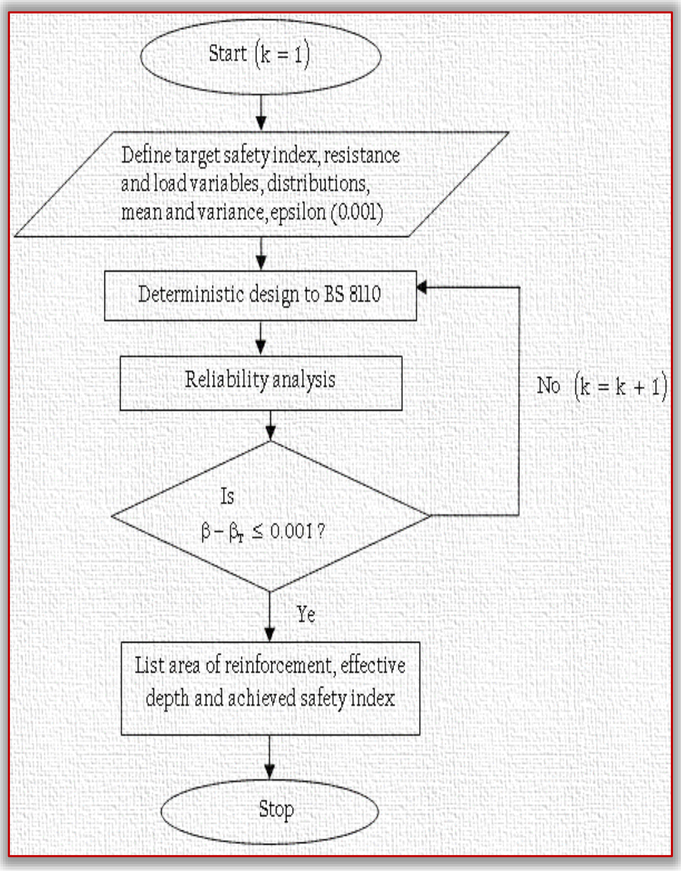

Figure 2: Flowchart showing the reliability based design procedure

Table 1: Statistics of the basic variables

| S/N | Random variable | Mean | Standard Deviation | Coefficient of Variation | Type of Probability Distribution |
|---|---|---|---|---|---|
| 1 | $Q_{KC}$ | 65,000N | 1950N | 0.030 | Gumbel |
| 2 | $Q_{KB}$ | 3.25N/mm | 0.975N/mm | 0.30 | Gumbel |
| 3 | $K_{mod}$ | 0.90 | 0.135 | 0.15 | Lognormal |
| 4 | L | 3000mm | 30mm | 0.01 | Normal |
| 5 | b | 300mm | 3mm | 0.01 | Normal |
| 6 | $f_{m,k}$ | 24N/mm$^2$ | 3.6N/mm$^2$ | 0.15 | Lognormal |
| 7 | $f_{c,k}$ | 21N/mm$^3$ | 3.15N/mm$^3$ | 0.15 | Lognormal |
| 8 | $\gamma_m$ | 1.30 | 0.195 | 0.15 | Lognormal |
| 9 | $\alpha_C$ | Varying | Varying | 0.30 | Fixed |
| 10 | $\alpha_L$ | Varying | Varying | 0.30 | Fixed |

Source: (Benu and Sule, 2012)

## RESULTS OF THE PROBABILITY-BASED DESIGN

The results of the probability-based design of a pin-ended solid square timber column of strength class C24 obtained at different values of target safety indices are presented in table 1 to 4 respectively.

The axial and lateral load ratios were varied for each performance criterion in the design process while other relevant parameters were kept constant. The values of the cross sectional dimension that satisfy the specified safety index, the coefficient of variation of the column and load ratio was determined. The design was found to be adequate at load ratio value of 0.2, 0.3, 0.4 and 0.5 respectively, for the two performance criteria considered as the value of the implied safety index and the value of the predefined target safety index coincided (Eurocode 0, 2002).

From Table 1 to 4, it can be seen that economical cross sectional dimensions were obtained at lower target safety index. It can also be seen that increase in load ratio (both in axial compression and bending) and coefficient of variation produces a corresponding increase in the cross sectional dimensions of the column.

The results obtained showed that the results obtained due to failure of the column in axial compression are more economical in terms of material consumption than those obtained due to column failure in bending.

Table 1: Column dimension for axial and lateral load ratio of 0.20

| Failure criteria | Target Safety Index | Coefficient of variation of column strength in compression and bending | | |
|---|---|---|---|---|
| | | 10 % | 15% | 20% |
| Compression | 2.0 | 125 | 132 | 143 |
| | 2.5 | 135 | 147 | 163 |
| | 3.0 | 150 | 169 | 195 |
| | 3.5 | 167 | 193 | 241 |
| | 4.0 | 189 | 229 | 320 |
| Bending | 2.0 | 175 | 180 | 190 |
| | 2.5 | 186 | 196 | 208 |
| | 3.0 | 201 | 215 | 234 |
| | 3.5 | 217 | 237 | 270 |
| | 4.0 | 240 | 265 | 323 |

Table 2: Column dimension for axial and lateral load ratio of 0.30

| Failure criteria | Target Safety Index | Coefficient of variation of column strength in compression and bending | | |
|---|---|---|---|---|
| | | 10 % | 15% | 20% |
| Compression | 2.0 | 129 | 138 | 148 |
| | 2.5 | 140 | 153 | 169 |
| | 3.0 | 158 | 175 | 204 |
| | 3.5 | 173 | 201 | 250 |
| | 4.0 | 196 | 245 | 340 |
| Bending | 2.0 | 180 | 185 | 195 |
| | 2.5 | 192 | 201 | 213 |
| | 3.0 | 207 | 220 | 240 |
| | 3.5 | 223 | 242 | 276 |
| | 4.0 | 245 | 272 | 340 |

Table 3: Column dimension for axial and lateral load ratio of 0.40

| Failure criteria | Target Safety Index | Coefficient of variation of column strength in compression and bending | | |
|---|---|---|---|---|
| | | 10 % | 15% | 20% |
| Compression | 2.0 | 134 | 143 | 153 |
| | 2.5 | 145 | 158 | 175 |
| | 3.0 | 162 | 182 | 215 |
| | 3.5 | 180 | 208 | 260 |
| | 4.0 | 205 | 255 | 365 |
| Bending | 2.0 | 185 | 190 | 200 |
| | 2.5 | 196 | 205 | 218 |
| | 3.0 | 213 | 226 | 247 |
| | 3.5 | 228 | 248 | 282 |
| | 4.0 | 250 | 282 | 350 |

Table 4: Column dimension for axial and lateral load ratio of 0.50

| Failure criteria | Target Safety Index | Coefficient of variation of column strength in compression and bending | | |
|---|---|---|---|---|
| | | 10 % | 15% | 20% |
| Compression | 2.0 | 137 | 147 | 158 |
| | 2.5 | 150 | 163 | 181 |
| | 3.0 | 168 | 190 | 220 |
| | 3.5 | 185 | 215 | 267 |
| | 4.0 | 210 | 262 | 280 |
| Bending | 2.0 | 188 | 195 | 202 |
| | 2.5 | 200 | 210 | 223 |
| | 3.0 | 216 | 230 | 255 |
| | 3.5 | 233 | 253 | 288 |
| | 4.0 | 256 | 286 | 347 |

However, the values of the cross sectional dimensions produced at varying values of load ratios are all found to be adequate as they satisfied the two performance criteria.

## CONCLUSION

A computer program developed in MATLAB language has been developed to automate the probabilistic design of a pin-ended solid timber column under axial compression and bending designed in accordance with EC-5 (1995) design rules.

The design point of the First Order Reliability method was invoked to check the safety level of the designed section. It was shown that the results obtained due to failure of the column in axial compression are more economical in terms of material consumption than those obtained due to column failure in bending. It was also shown that higher column cross sectional dimensions were obtained at higher safety index. However, the values of the cross sectional dimensions produced at varied load ratios of 0.2, 0.3, 0.4 and 0.5 all satisfied the performance criteria in axial compression and bending.

## References

[1] Abejide, O. S. Incorporation of advanced second moment reliability assessment method into assessment and design. World Journal of Engineering and Pure and Applied Science, 2 (4), pp. 112-117, 2012.

[2] Abubakar, I. Reliability analysis of structural design parameters of reinforced concrete strip footings. Journal of Applied Science and Research, 2 (7), pp. 397-401, 2006.

[3] Afolayan, J.O. Parameter selection for safe design under uncertain conditions: A truss example. Journal of Engineering and Applied Science, 21, pp. 77-82, 2002.

[4] Afolayan, J.O. Probability based design of glued thin-webbed timber beams. Asian Journal of Civil Engineering (Building and Housing), Vol.6, Nos. 1-2, pp. 75-84, 2005

[5] Afolayan, J.O and Abubakar, I. Reliability analysis of reinforced concrete one-way slabs: The ultimate conditions. Nigerian Journal of engineering, 11(2), pp. 28-31, 2003.

[6] Benu, M.J. and Sule, S. Probabilistic failure analysis of a solid timber column. Nigerian Journal of Tech. (NIJOTECH), Vol. 31, No.3, pp. 405-408, 2012.

[7] Sule, S. and Benu, M.J. Reliability analysis of a solid timber column subjected to axial and lateral loading. FUOYE Journal of Engineering and Technology, 4 (2), pp. 1-5, 2019.

[8] El-Reedy, M. Reinforced concrete structural reliability. Taylor and Francis, Boca Raton, FL, 2013.

[9] Ogork, E.N. and Nakore, A.K. Reliability analysis of a two span floor designed according to Eurocode 2. Nigerian Journal of Technology (NIJOTECH), 36 (1), pp. 18-25, 2017.

[10] Eurocode 0. Eurocode-basis of structural design, British Standards, CEN, Brussels, 2002.

[11] Eurocode 5. Design of timber structures part 1.1. General rules and rules for buildings. Publication of European Committee for Standardization, 1995.

[12] Melchers, R. E. Structural Reliability Analysis and Prediction. Second Edition, John Wiley and Sons, Chichester, England, 1999.

[13] Ranganathan, R. Structural Reliability, Analysis and Design, Jaico Publishing House, Mumbai, 1990.

[14] Salisu, D., Abubakar, I., Ejeh, S.P. and Afolayan, J.O. (2009). Probabilistic design of mechanically-jointed timber beams using Eurocode 5. Indian Journal of Science and Technology, 2 (10), pp. 62-65, 2009

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