1. Tamás SZŐCS, 2. Zoltán Bálint SUSI

THERMOELASTIC PROBLEM OF MULTILAYERED CURVED BEAMS

1,2 Institute of Applied Mechanics, University of Miskolc, Miskolc, HUNGARY

Abstract: The main objective of this paper is to determine the thermal stresses and displacements in multilayered curved beams subjected to thermal loading and concentrated moment. An analytical solution is presented to tackle this thermoelastic problem of multilayered circular arc with constant radius. The model can be modified to determine the stresses within radially graded curved beams. The developed plane stress method is compared to result coming from finite element simulations. Our main focus is to determine the analytical solution for the stresses and displacement within the beam which is subjected to constant temperature field and concentrated moment at the end of the beam.
Keywords: curved beam, multilayered, thermal stresses

INTRODUCTION

Curved beams are frequently used components in frame structures. Recent years a lot of studies have been performed on the mechanics of multilayered and functionally graded beams from different aspects. A lot of books deal with the stress analysis of beams, such as [1-7]. Bimetal components are one of the applications for multilayered beams. Several works have dealt with the mechanics of bimetallic strips which consist of two different components [8–12] although curved beams were not considered.

Papers such as [13-16] deal with the stability problem of curved beams subjected to only mechanical loading. These articles present an analytical method to the non-linear stability investigation of curved beams. The model can be used not only for homogeneous materials but also for functionally graded distributions. In [15] the mechanical load is a concentrated radial force at the crown point, while in [16], it is exerted in the small vicinity of the crown. Furthermore, in [13] the application point can be arbitrary along the centroidal axis. There are several papers (e.g. [17-19]) where multilayered structures were analysed and used to approximate simple radially graded components, such as disks or spherical bodies.

In paper [18] an analytical solution was developed for spheres using Boussinesq displacement potentials, in which the functions of the material properties – except the Poisson's ratio - are power law functions, in paper [19] the problem is solved as the superposition of the simpler subproblem. Then these multilayered methods were used to calculate stresses in radially graded spheres.

Pydah and Sabale [20] solved the flexure problem of bi-directional functionally graded circular beams subjected to various tip loads. Eslami et. al [21] used a two-step perturbation technique to present the solution of functionally graded shallow tube subjected to lateral pressure and temperature field, where the properties of the arch were distributed through the radial direction using a specific power law function.

In paper [22] a method is presented to calculate the stresses and displacements in bimetal curved beam subjected to only constant temperature field based on strength of materials and plane stress theories. In the work [23] the solution of a radially graded circular curved beam is presented, where the modulus of elasticity varies according to a power law function. Ecsedi and Baksa [24] presented a method to calculate the stresses in circular polar orthotropic beams subjected to radial loading.

A multilayered curved beam is considered. The number of layers is denoted by n, the homogeneous layers are perfectly bonded. A cylindrical coordinate system will be used, the sketch of the problem - for four layers, as an example - can be seen in Figure 1. Our main focus is to determine the analytical solution for the stresses and displacement within the beam which is subjected to constant temperature field (T) and concentrated moment at the end of the beam. The internal radius of the i-th layer is denoted by R_i, the outer radius is R_{i+1}. The symmetry plane of the problem is the z=0 plane.

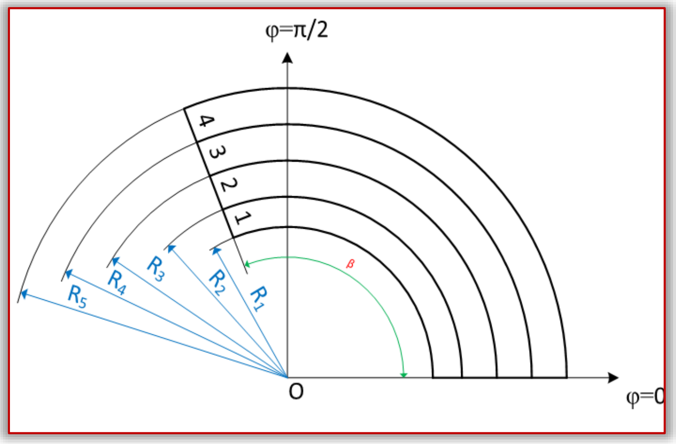


Figure 1. The sketch of the problem in case of 4 layers

GOVERNING EQUATIONS

In the basis of the plane stress model, the boundary conditions of the considered problem can be expressed as

$$\sigma_r(R_1,\varphi)=\sigma_r(R_{n+1},\varphi)= =\tau_{r\varphi}(R_1,\varphi)=\tau_{r\varphi}(R_{n+1},\varphi)=0,\quad 0\le\varphi\le\beta, \tag{1}$$

$$\sigma_\varphi(r,\beta)=\sigma_\varphi(r,0)= =\tau_{r\varphi}(r,\beta)=\tau_{r\varphi}(r,0)=0,\quad \sigma_\varphi(r,\beta)=\sigma_\varphi(r,0)=0,\quad R_1\le r\le R_{n+1}, \tag{2}$$

furthermore the following weak form equations will be satisfied:

$$N=\int_{R_1}^{R_{n+1}}\sigma_\varphi(r,\beta)\mathrm{d}A=\int_{R_1}^{R_{n+1}}\sigma_\varphi(r,0)\mathrm{d}A=0, \tag{3}$$

$$M=\int_{R_1}^{R_{n+1}}r\sigma_\varphi(r,\beta)\mathrm{d}A=\int_{R_1}^{R_{n+1}}r\sigma_\varphi(r,0)\mathrm{d}A. \tag{4}$$

In the previous equations σ_r and σ_φ, are the radial and tangential normal stresses. According to the Euler-Bernoulli beam theory, the displacement field can be given as:

$$u_i(r,\varphi)=U_i(r)+f_1\cos\varphi+f_2\sin\varphi,\quad v_i(r,\varphi)=Dr\varphi+f_1\sin\varphi-f_2\cos\varphi+f_3r,\quad (i=1\ldots n), \tag{5}$$

where u and v denote the radial and tangential displacement components. From the kinematic boundary conditions the unknown f and D constants can be determined. For example we can use:

$$u_{ri}(R_1,0)=0,\ v_1(R_1,0)=0,\ v_2(R_n,0)=0. \tag{6}$$

The kinematic equations yield

$$\varepsilon_{\varphi i}=\frac{U_i}{r}+D,\ \varepsilon_{ri}=\frac{\mathrm{d}U_i}{\mathrm{d}r},\ i=1\ldots n, \tag{7}$$

where ε_{ri} and $\varepsilon_{\varphi i}$ denote the normal strain coordinates of the strain tensor in the i-th layer. The stress-strain relation for the i-th layer can be expressed as

$$\varepsilon_{ri}=E_i^{-1}(\sigma_{ri}-\nu_i\sigma_{\varphi i})+\alpha_iT, \tag{8}$$

$$\varepsilon_{\varphi i}=E_i^{-1}(\sigma_{\varphi i}-\nu_i\sigma_{ri})+\alpha_iT,\quad (i=1\ldots n). \tag{9}$$

The combination of Eqs. (8) and (9) with the compatibility condition leads to

$$r\frac{\mathrm{d}\varepsilon_{\varphi i}}{\mathrm{d}r}+\varepsilon_{\varphi i}-\varepsilon_{ri}-D=0,\quad (i=1\ldots n),\quad r\frac{\mathrm{d}}{\mathrm{d}r}(-\nu_i\sigma_{ri}+\sigma_{\varphi i}+E_i\alpha_iT)- -(1+\nu)\sigma_{ri}+(1+\nu)\sigma_{\varphi i}-E_iD=0. \tag{10}$$

From the equilibrium equation we get

$$\frac{\mathrm{d}\sigma_{ri}}{\mathrm{d}r}+\frac{\sigma_{ri}-\sigma_{\varphi i}}{r}=0,\quad \sigma_{\varphi i}=r\frac{\mathrm{d}\sigma_{ri}}{\mathrm{d}r}+\sigma_{ri},\quad (i=1\ldots n). \tag{11}$$

The combination of the previous equations yields the following differential equation for the different layers

$$r\frac{\mathrm{d}}{\mathrm{d}r}((1-\nu_i)\sigma_{ri}+r\frac{\mathrm{d}\sigma_{ri}}{\mathrm{d}r}+E_i\alpha_iT)- -(1+\nu)\sigma_{ri}+(1+\nu)\left(r\frac{\mathrm{d}\sigma_{ri}}{\mathrm{d}r}+\sigma_{ri}\right)-E_iD=0. \tag{12}$$

The solution of the differential equation can be expressed as

$$\sigma_{ri}(r)=C_{i1}+\frac{C_{i2}}{r^2}+\frac{E_iD}{2}\ln r,\quad \sigma_{\varphi i}(r)=C_{i1}-\frac{C_{i2}}{r^2}+\frac{E_iD}{2}(\ln r+1), \tag{13}$$

where the number of unknown constants is $2n+1$. The boundary and fitting conditions of the multilayered beam (with thickness b) are:

$$\sigma_{r1}(R_1)=\sigma_{rn}(R_n)=0,\ \sigma_{ri}(R_i)=\sigma_{ri}(R_i),\quad U_1(R_i)=U_2(R_i),\ (i=1\ldots n), \tag{14}$$

$$\sum_{i=1}^{n}\int_{R_i}^{R_{i+1}}r\sigma_\varphi\mathrm{d}r=\int_{R_i}^{R_{n+1}}r\sigma_\varphi\mathrm{d}r=Mb^{-1}. \tag{15}$$

With system of equations (14), (15) we can calculate the unknown constants for the different layers. The stress distribution does not depend on the angular coordinate.

This method can be modified to calculate stresses within radially graded beams subjected to constant temperature and concentrated moment. In this case we need n homogeneous layers and we have to discretize the values of the material properties for the different layers. One technique is to use the value of the function of the material parameters at the middle of the layer R_{mi} [19]

$$E_i=E(r=R_{mi}),\nu_i=\nu(r=R_{mi}),\quad \alpha_i=\alpha(r=R_{mi}),\ i=1\ldots n. \tag{16}$$

The more layers we use the more accurate the solution is.

NUMERICAL EXAMPLES

In our first numerical examples the following numerical data will be used:

$$R_1=1\text{m},\ R_2=1.05\text{m},\ R_3=1.09\text{m},\ R_4=1.14\text{m},$$
$$E_1=69\text{ GPa},\ E_2=200\text{ GPa},\ E_3=69\text{ GPa},$$

$$\nu_1 = 0.33,\ \nu_2 = 0.3,\ \nu_3 = 0.33,\ \alpha_1 = 2.2 \cdot 10^{-5}\frac{1}{\mathrm{K}},$$
$$\alpha_2 = 1.1 \cdot 10^{-5}\frac{1}{\mathrm{K}},\ \alpha_3 = 2.2 \cdot 10^{-5}\frac{1}{\mathrm{K}},\ T = 150°\mathrm{C},$$
$$\beta = 290°.$$

The presented method is compared to results coming from finite element simulations. We used Maple and Abaqus CAE to carry out the calculations. The deformed mesh of the first problem can be seen in Figure 2. Quadratic coupled temperature-displacement elements were used to formulate the steady-state plane stress model [33].

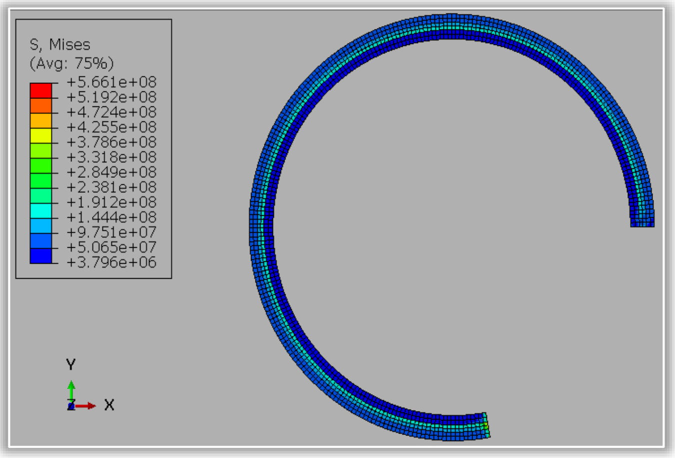


Figure 2. The finite element model of the first problem with the von Mises stress

In Figure 2 we can see that the stress distribution does not depend on the angular coordinate except at the ends of the beam. The stress distributions are plotted in Figure 3 in case of pure constant temperature loading.

In the next case we apply an additional concentrated moment (5 kNm) at the free end of the curved beam. The results can be seen in Figure 4.

In the second numerical example the following numerical data will be considered:

$$R_1 = 0.03\mathrm{m},\ R_2 = 0.035\mathrm{m},\ R_3 = 0.04\mathrm{m},$$
$$R_4 = 0.045\mathrm{m},\ R_5 = 0.05\mathrm{m},\ E_1 = 117\ \mathrm{GPa},$$
$$E_2 = 200\ \mathrm{GPa},\ E_3 = 69\ \mathrm{GPa},\ E_4 = 117\ \mathrm{GPa},$$
$$\nu_1 = 0.355,\ \nu_2 = 0.3,\ \nu_3 = 0.33,\ \nu_4 = 0.355,$$
$$\alpha_1 = 1.62 \cdot 10^{-5}\frac{1}{\mathrm{K}},\ \alpha_2 = 1.1 \cdot 10^{-5}\frac{1}{\mathrm{K}},$$
$$\alpha_3 = 2.2 \cdot 10^{-5}\frac{1}{\mathrm{K}},\ \alpha_4 = 1.62 \cdot 10^{-5}\frac{1}{\mathrm{K}},\ T = 150°\mathrm{C},$$
$$\beta = 120°.$$

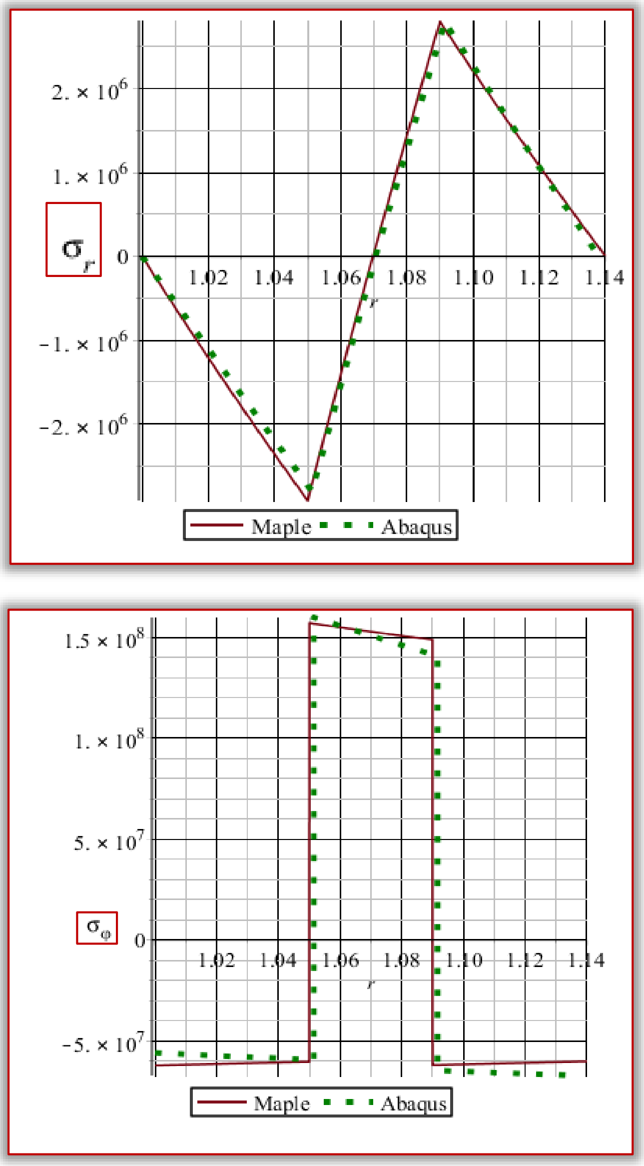
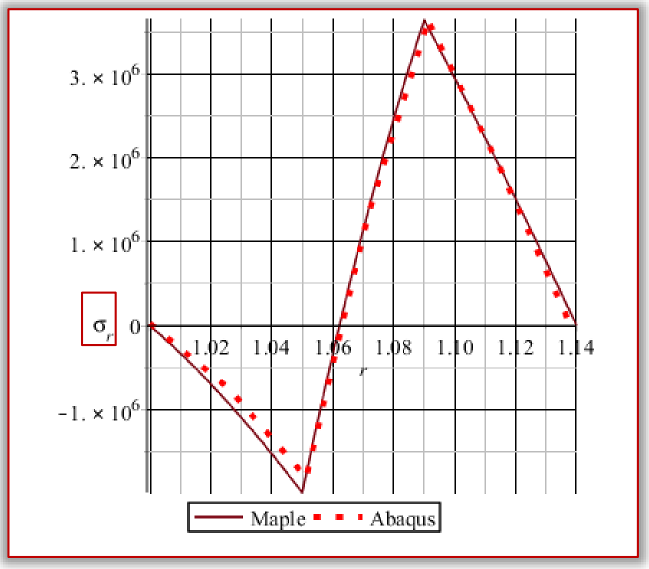


Figure 3. The normal stresses of the first problem subjected to constant temperature field



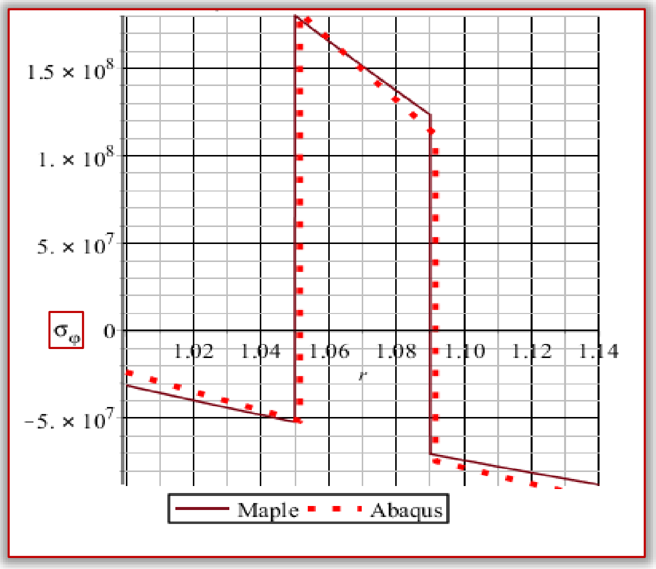


Figure 4. The normal stress distribution of the first problem with constant temperature and concentrated moment

The number of layers is four, the geometry is more strip-like and the mesh of the problem is shown in Figure 5.

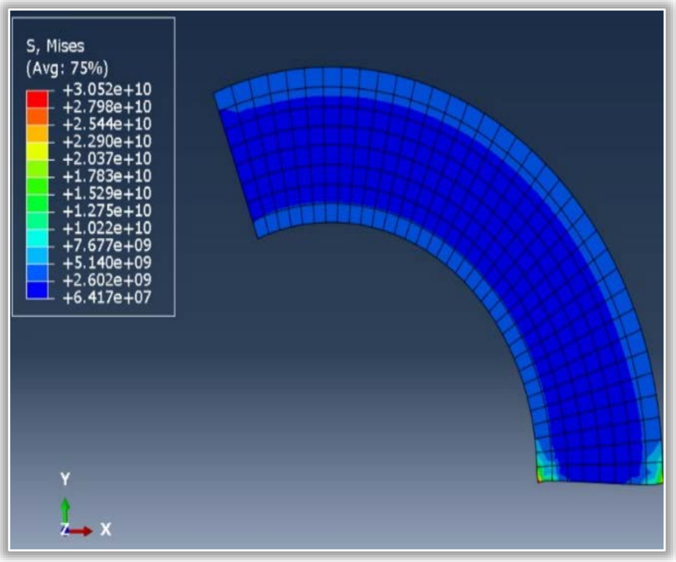


Figure 5. The mesh of the second problem with the von Mises stress distribution

Figure 6 shows the normal stress distribution of the curved beam subjected to constant temperature field while in Figure 7 we can see the effect of an additional moment M=5 kNm on the stress distribution.

The results are in good agreement and we verified that the stress field of these multilayered curved beams with constant curvature depends only on the radial coordinate except in the vicinity of the end cross sections.

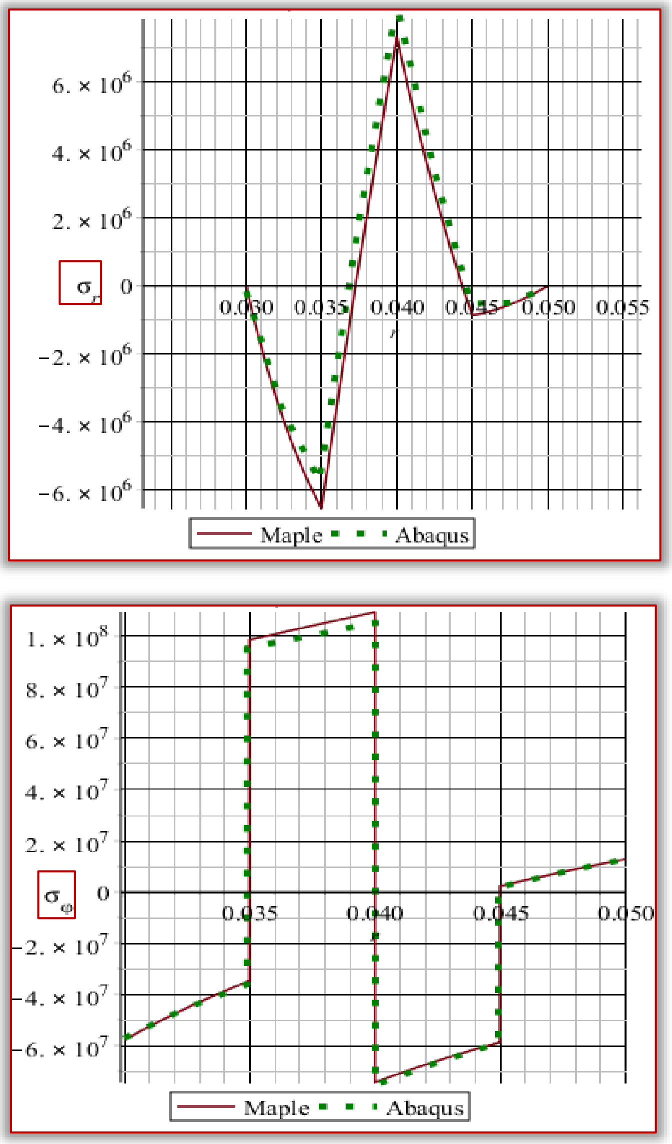
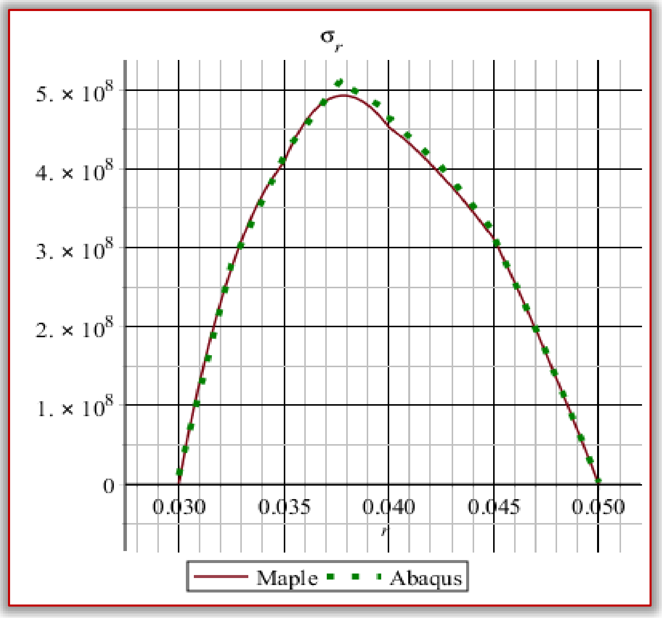


Figure 6. The normal stresses of the first problem with constant temperature field



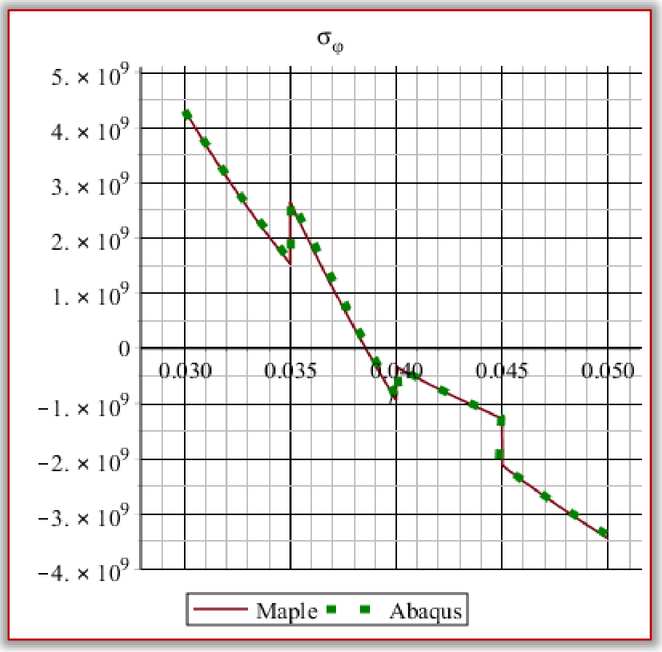


Figure 7. The normal stress distributions of the second problem with constant temperature and concentrated moment

SUMMARY

An analytical method was presented to determine the stresses and displacement in multilayered curved beam subjected to specific thermal and mechanical loads. The curvature of the beam was constant.

The developed method is based on the elasticity equations of plane stress state and Euler Bernoulli beam theory.

A method is presented to tackle the problem of radially graded beams subjected to constant temperature and concentrated moment.

The results were compared to solutions coming from finite element simulations.

References

[1] Timoshenko. S. P.: Analysis of bi-metal thermostats. Journal of the Optical Society of America, 11(3), pp. 233–255, 1925.

[2] Timohsenko. S. P.: The Collected Papers. McGraw Hill, New York, 1953

[3] Young W. C., Budynas R. G.: Roark's Formulas of Stress and Strain. 7th ed. McGraw Hill, New York, 2002.

[4] Boley, B. A. and Weiner, J. H.: Theory of Thermal Stresses. John Wiley & Sons Inc., New York, 1960.

[5] Carreci, E, Fazzolari, F, Cinefra, M.: Thermal Stress Analysis of Composite Beams, Plates, Shells. Academic Press, New York, 2017.

[6] Nowinski I.L.: Theory of Thermoelasticity with Applications. Sythoff and Noordhoff, Alpen aan den Rijn, 1978.

[7] Lekhnitskii, S. G.: Theory of Elasticity of an Anisotropic Body. Mir Publishers, Moscow, 1981.

[8] D. Ramos, I. Mertens, M. Callega, and I. Tamayo.: Study on the origin of bending induced by bimetallic effect on microcantilever. Sensors, 7(9), pp. 1757–1765, 2007.

[9] Rao, A. V., Prasad K. S. V., Avinash, M., Nagababu, K., Manohar, V., Raju, P. S. R. and Chandra, G. R.: A study on deflection of a bimetallic beam under thermal loading using finite element analysis. International Journal of Engineering and Advanced Technology, 2(1), pp. 81–82 2012.

[10] Aignatoaie. M.: FEA study on the elastic deformation process of a simple bimetal beam. Applied Mechanics and Materials, 371(8), pp. 448–452, 2013.

[11] Suhir. E.: Interfacial stresses in bimetal thermostats. Journal of Applied Mechanics, 56(3), pp. 595–600, 2013.

[12] Ecsedi, I., Gönczi, D.: Thermoelastic stresses in nonhomogeneous prismatic bars. Annals of Faculty of Engineering Hunedoara - International Journal of Engineering, 13(2), pp. 49-52, 2015.

[13] Kiss, L. P.: Nonlinear stability analysis of FGM shallow arches under an arbitrary concentrated radial force. International Journal of Mechanics and Materials in Design, 16(1), pp. 91-108, 2020.

[14] Babaei, H., Kiani, Y., Eslami, M.R.: Thermomechanical non-linear in-plane analysis of fix-ended FGM shallow arches on nonlinear elastic foundation using two-step perturbation technique. Int. J. Mech. Mater. Des. 15(2), 225–244, 2019.

[15] Kiss, L.P.: In-plane Buckling of Rotationally Restrained Heterogeneous Shallow Arches Subjected to a Concentrated Force at the Crown Point. Journal of Computational and Applied Mechanics, 9(2), pp. 171-199, 2014.

[16] Kiss, L. P.: Sensitivity of FGM shallow arches to loading imperfection when loaded by a concentrated radial force around the crown. International Journal of Non-linear Mechanics, 116, pp. 62-72, 2019.

[17] Gönczi, D.: Analysis of Rotating Functionally Graded Disks with Arbitrary Material Properties. Acta Technica Corviniensis – Bulletin of Engineering, 11, pp. 1-6, 2018.

[18] Chenyi Zheng, Xiaobao Li, Changwen Mi.: Reducing stress concentrations in unidirectionally tensioned thick-walled spheres through embedding a functionally graded reinforcement. International Journal of Mechanical Sciences, 152, pp. 257-267, 2019.

[19] Gönczi, D.: Thermoelastic analysis of thick-walled functionally graded spherical pressure vessels with temperature-dependent material properties. Journal of Computational and Applied Mechanics, 12(2), pp. 109-125, 2017.

[20] Pydah A., Sabale, A.: Static analysis of bi-directional functionally graded curved beams. Composite Structures 160: 867–876, 2017.

[21] Babaei, H, Kiani, Y, Eslami M.R.: Geometrically nonlinear analysis of functionally graded shallow curved tubes in thermal environment. Thin-walled Structured 123, pp. 48-57, 2018.

[22] Gönczi, D.: Analysis of curved bimetallic beam. Journal of Computational and Applied Mechanics, 41(1-2), pp. 41-51, 2019.

[23] Haskul, M.: Elastic state of functionally graded curved beam on the plane stress state subject to thermal load. Mechanics Based Design of Structures and Machines, 47., 2019.
[24] Ecsedi I., Baksa A.: A half circular beam bending by radial loads. Journal of Computational and Applied Mechanics, 12(1), pp. 3-18., 2017.
[25] Abaqus Standard User's Manual Version 6.13, 2017.

ACTA TECHNICA CORVINIENSIS – Bulletin of Engineering
ISSN: 2067-3809