[1.]Norge Isaías COELLO–MACHADO, [2.]Elke GLISTAU

APPLICATION OF EXPERIMENTAL DESIGN IN LOGISTICS SYSTEMS

[1.] Universidad Central "Marta Abreu" de Las Villas, Santa Clara, CUBA
[2.] Otto von Guericke University, Magdeburg, GERMANY

Abstract: Design of Experiments (DoE) is a well–known method in quality management but seldom used in the logistics area. An important question of science is to find the real factors to control complex processes and systems and to know their levels and influences to control all processes and systems in a right manner. To find the optimum in control there are many experiments necessary – practical and theoretical ones. Here the sensitivity analysis as well as simulation and Design of Experiments are used. That is why this paper gives an overview about the basics and ideas of Design of Experiments. An example of practical use is given, showing how a special logistics system (a sorter), can be optimized. The problem is to minimize the failure rate of not identified items with the help of the two factors speed of the sorter and distance between the packages. The experiment should answer several questions and reduce the costs for optimisation of the logistics system. The paper shows all relevant steps to solve the problem and gives some useful hints for applying DoE procedure to other similar problems.
Keywords: Design of Experiments, logistics systems, sorter, optimisation

INTRODUCTION

DoE has an old tradition and history. Famous names in this context are Fisher, Taguchi and Shainin. DoE is a structured and established method of quality management. (Fig. 1, Compare [1] [2]) The goal is to realize as few experiments as possible to minimize optimisation costs. DoE requires only a small set of experiments and thus helps to reduce these costs.

The Shainin methods (Compare [3] [4]) include the full factorial ones. These are typical methods of classic statistical research methods. DoE determines the relationship between the different important factors (x_i) affecting a process or system and the output (result) of that process or system (y). DoE has significantly more to offer than a "one change at a time experimental method".

A "one change at time experimental method" has always the risk that the researcher finds only the most significant effect on the output. DoE also focuses on dependency and interactions between the most important factors. DoE first plans for all possible dependencies and interactions, then it describes exactly which data are necessary to assess them.

The main objective in the practice of experimental statistical design is to obtain the necessary information with the least amount of costs and the greatest rapider possible.

The exact length and size of the experiment are set by the experimental design before the real experiments begin. DoE involves designing a set of experiments, in which all relevant factors are varied systematically. When the results of these experiments are analysed, they help to identify optimal conditions.

Other results of DoE are the factors that mainly influence the results (high effect), the factors that have little influence on the results (small effects), as well as the detection of interactions and synergies between the factors.

The common way to realize DoE is the following one:

Define the objective of the investigation.
Define the variables (factors) y_k that will be measured to describe the output.
Define the variables (factors) that will be controlled x_i during the experiment.
Define the ranges of variation and the factor levels of each factor.
Define and optimise the experimental plan.
Prepare and carefully carry out the experiments and secure the results.
Do the statistical analysis and interpret the results.
Use the knowledge to optimise the process or system.

This is only a short introduction in DoE. Use the literature in [5] and [6] for further information about the basics of DoE, especially the full–factorial ones. DoE is used only seldom in the logistics area because it is real little unknown in this field.

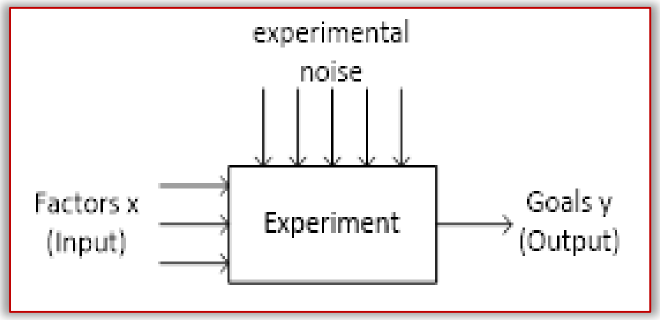

Figure 1.Simple model of DoE

PRACTICAL PROBLEM

Now in this case, a logistical system is being investigated, the sorter, to give a practical example of how to realize DoE in logistics. A sorter is a complex and modular logistical technical system that includes the logistical sorting process. The task of the sorter is to place small parcels, packages, fragile and sensitive products as well as heavy freight goods, boxes and baggage safely onto a conveyor. After that, it is necessary to discharge them into the right destination. The main objective of a sorter system is to fast sort items. This should been realized by using a big quantity of items, a high efficiency of the sorter process and a short throughput time of items. (Hundred percent of items and other objects should be sorted at the very first time.)

To calculate the efficiency rate, the amount of packages or other items is used. It describes the items, which fulfil more than one circulation in the sorter system. This rate is marked as the failure rate of the system.

On one side the main objective is to minimize the amount of packages, with more than one circulation in the sorter systems, and on the other side as many items as possible should be sorted to achieve a high efficiency.

DESIGN ON EXPERIMENTS

Often staff responsible for the optimisation of the logistics systems believe, that there is a great number of relevant factors, but really only two factors are the most important ones. These are:

- # Factor 1: Speed of the sorter
- # Factor 2: Distance between the packages

This is a classical 2–factor–problem with;

$$k = 2 \text{ factors} \tag{1}$$

The staff is convinced that a high speed and a small distance between the packages is good to realize a high efficiency of the sorter system. How will this change the failure rate? Will the failure rate increase with higher speed and smaller distance? The experiments should help to answer several questions:

- # How many experiments are necessary to analyse the problem?
- # How many failures will happen at each combination of factors?
- # How big is the influence of speed and distance on the failure rate?
- # Is the influence of both factors speed and distance significant or not?
- # Which combination of speed and distance will achieve the smallest failure rate?
- # Are there visible limits of the sorter?
- # How many failures should been accepted in total?

SOLUTION

Each factor gets two levels: a low level and a high one. The low level gets the value (–1). The high level gets the value (+1).

Two factors (k) and two factor levels (p) come to four possible combinations in summary.

$$m=4 \tag{2}$$

Moreover, each combination should be realized two times to make a statistical analysis possible. The task is to get more information on which combination is the best.

The number of necessary repetitions n.

$$n=2 \tag{3}$$

There will be three repetitions, but that is one repetition cycle more than statistical necessary. Therefore, we will get three results in each combination (y_1, y_2, y_3).

We use the number of repetitions n.

$$n=3 \tag{4}$$

The interaction the both factors is signed by x_1x_2. It is calculated as the product of x_1 and x_2.

Table 1. Results of experiments y_1, y_2, and y_3

No.	x_1	x_2	x_1x_2	y_1	y_2	y_3
1	–1	–1	1	7.6	7.9	7.8
2	1	–1	–1	13.2	13.6	13.9
3	–1	1	–1	4.3	4.1	4.3
4	1	1	1	9.4	10.0	10.1

The results are listed in table 2. It is important to mention that the sequence of experiments should be pure chance.

The sequence of experiments must be completely random. Random numbers can be generated or simply an urn can be used.

Table 2. Calculation of the arithmetic mean and the formula

No.	Y	Yx_1	Yx_2	Yx_1x_2
1	7.8	–7.8	–7.8	7.8
2	13.6	13.6	–13.6	–13.6
3	4.2	–4.2	4.2	–4.2
4	9.8	9.8	9.8	9.8
Sum	35.40	11.40	–7.27	–0.20
Sum/4	**8.85**	**2.85**	**–1.82**	**–0.05**

The first result is the following formulas (Compare Table 2):

$$b_0 = (\Sigma Y)/m=2,85 \tag{5}$$

$$b_1 = (\Sigma x_1{*}Y)/m \quad =2.85 \tag{6}$$

$$b_2 = (\Sigma x_2{*}Y)/m \quad = (-1.82) \tag{7}$$

$$b_{12} = (\Sigma x_1{*}x_2{*}Y)/m = (-0.05) \tag{8}$$

$$y=8.85+2.85x_1-1.82x_2-0.05x_1x_2 \tag{9}$$

Of the results of Main, effects and interactions can be interpreted:

- # The best combination with the lowest failure rate is the combination of speed and distance [(–1) (1)]. That means that a slow speed and a big distance is good for the results, for a small failure rate.

The influence of speed (2.85) is higher than the influence of distance (1.82) is.

The higher the speed x_1 the higher the failure rate y.

The bigger the distance x_2 the smaller the failure rate y.

— **Analysis of variance (ANOVA). Test of homogeneity of variance**

Y and the variance s_i^2 are used to calculate the variance homogeneity. This is an important test to check the DoE in general.

The main idea is that the variances of the experimental results are equal at all combinations of factors, because there were only the levels of factors are changed.

The formula is:

$$H_0: s^2 = s_1^2 = s_2^2 = s_3^2 = s_4^2 \tag{10}$$

Should this hypothesis be true, so the planning and the realization of experiments are also both true. Following the next values can be analyzed.

Otherwise, the planning or the realization of experiments is wrong. That means important factors are ignored. The conditions or other factors were not at a constant level during the experimental phase. In summary so we have wrong conditions at whole. It is possible that now more experiments are necessary to realize.

We use the formula 11.

If

$$(s_{max} / s_{min}) \leq F_{f1,f2,95\%} \tag{11}$$

then H_0 is true, otherwise H_0 is wrong

With

$$f_1 = f_2 = n - 1 = 3 - 1 = 2 \tag{12}$$

Because the thesis H_0 (compare formula 13) is true the experiment design is true too. Now the quantities can be analyzed.

$$(s_{max} / s_{min}) = 10.75 \leq F_{f1,f2,95\%} = F_{2,2,95\%} = 19.00 \tag{13}$$

— **Test of significance of each single factor and of interactions**

Now the test of significance of each effect is done. Therefore, we need the number of experiments (N) with the formula

$$N = m * n \tag{14}$$

The values are now calculated

$$S_d = \sqrt{\left(\frac{m}{N}\right) S^2} \tag{15}$$

with $s^2 = (\Sigma si^2)/m = 0.159$

Then the degrees of freedom are calculated

$$f = N - m \text{ and } t_{f,\,95\%} = t_{8,95\%} \tag{16}$$

At last will be calculated

$$t_{f,\,95\%} * S_d = 0.296 \tag{17}$$

The test gives the following results:

The factor with the most influence on the results is speed (2.85). A higher speed leads to a higher rate of failures. The coefficient is significant because it is higher than [0.296].

The factor with less influence on the results is distance (–1.82). A smaller distance leads to a higher rate of failures. The coefficient is significant because the value [1.82] is higher than [0.296].

The existence of interactions and synergies between both factors (–0.05) is given. This is not significant for the output, because the coefficient level [0.05] is smaller than the significance level [0.296].

Now it is possible to simplify the function as following (Compare to formula 9):

$$y=8.85+2.85x_1-1.82x_2 \tag{18}$$

The end of the experiments is not achieved yet. The task is to optimize speed and distance. Therefore, further experiments are necessary.

— **Adequacy of statistical model**

Now we use the formula 18 to calculate the theoretical results of experiments y_p (Table 3).

We calculate the difference

$$(Y-y_p)^2 \tag{19}$$

Table 3. Calculation of the differences between practical experiments and theoretically based calculated values

Nr.	Y	y_p	$(Y-y_p)^2$
1	7.77	7.82	0.0025
2	13.57	13.52	0.0025
3	4.23	4.18	0.0025
4	9.83	9.88	0.0025
Sum	35.40	35.40	0.01
Sum/4	**8.85**	**8.85**	**0.0025**

This third test compares the formula 18 and the results of the experiments (Table 3). If the variability of the model is smaller than the practical experimental standard deviation, then the model can be accepted and be used further. (Formulas 20–24)

$$F_{f1,\,f2,Model} \leq F_{f1,f2,95\%} \tag{20}$$

$$f_1 = m-(k+1) = 4 - (2+1) = 1 \tag{21}$$

$$f_2 = m = 4 \tag{22}$$

$$F_{1,\,4,Model} \leq F_{1,4,95\%} \tag{23}$$

$$0.03 \leq 7.709 \tag{24}$$

Because the variability of the model (0.03) is smaller than the experimental standard deviation (7.709) the model can be accepted and be used further.

Here in necessary to keep in mind, that these results are valid only for the evaluated experimental interval and under the established experimental conditions.

It should be noted that even at the highest levels of development of modern industry, such as industry 4.0

and logistics 4.0 [7] the detailed study of the influence of factors on the response of a given variable continues to be of vital importance.

CONCLUSIONS

This paper gives an overview about the basics and ideas of Design of Experiments. The focus is on the full factorial experiments. An example of practical use is given. It shows how a special logistics system, a modular sorter, can be optimized by using DoE. The practical problem is to minimize the failure rate of not identified items. There are two factors especially important. These are the speed of the sorter and the distance between the packages. The experiment should answer several questions and reduce the costs for the optimisation of the logistics system. The paper shows all relevant steps to solve the problem and gives some useful hints to transfer the procedure to other similar problems.

References

[1] Glistau, E.; Coello Machado, N–I.; Illés, B: Logistics and Design of experiments. MicroCAD 2008. 03–20–2008 P szekció – Miskolc. ISBN 978–963–661–825–4, p. 29–36

[2] Glistau, E.; Coello Machado, N–I.: Design of experiments and six sigma methods in logistics. Advanced Logistic Systems. Miskolc. Bd. 2.2008. p. 47–54

[3] Illés, B.; Glistau, E.; Coello Machado, N–I.: Logistik und Qualitätsmanagement. ISBN 978–963–87738–1–4. 1. Edition Miskolc 2007. 196 pages. p. 70 – 74

[4] Illés, B.; Glistau, E.; Coello Machado, N–I.: Logística y Gestión de la Calidad. ISBN 978–963–358–007–3. 1. Edition Miskolc 2012. 194 pages. P. 70 – 74

[5] Montgomery, D.C.: Design and Analysis of Experiments. 8th ed. 2012. John Wiley Sons. P. 233–284. ISBN 978–1118146927

[6] Kleppmann, W.: Versuchsplanung – Produkte und Prozesse optimieren. Taschenbuch. 9th ed. 2016. Hanser Verlag München Wien. p. 101–127. ISBN 13 978–3–446–44716–5

[7] Glistau, E.; Coello Machado, N. I.: Industry 4.0, Logistics 4.0 and materials – chances and solutions. In: Novel trends in production devices and systems IV: NTPDS IV; Special topic volume with invited peer reviewed papers only – Zürich: Trans Tech Publications, S. 307–314, 2018;

ACTA TECHNICA CORVINIENSIS – Bulletin of Engineering
ISSN: 2067–3809