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ANALYSIS AND FORMING MECHANISM OF A NEW CHAOTIC ATTRACTOR

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Abstract: In this study, a new chaotic attractor is analyzed which is found by adding a quadratic nonlinearity to another chaotic attractor. First of all, time series analysis is performed for the new chaotic system. One of the methods to calculate the amount of chaos in a system is Lyapunov exponent which measure the exponential divergence of initially close state-space trajectories. Then, the conclusions are strengthened with analysis of the Lyapunov exponent and bifurcation diagram. Lastly, the forming mechanism of the new chaotic attractor is given with different values of a posted constant variable. In this study, the newfound chaotic attractor is investigated. Time series analysis, bifurcation diagram, and largest Lyapunov exponent decomposition of this new attractor are given for one of the constant parameters of it. This study has further given forming mechanism analysis of the attractor and sought the forming mechanism of it by explaining dynamical behaviors under varying values of a newly added constant parameter. Consequently, an exhaustive understanding is revealed for a new chaotic attractor through different ways.
Keywords: chaotic attractor, time series analysis, Lyapunov exponent, bifurcation diagram, forming mechanism

INTRODUCTION

There are characteristic features of chaotic systems which meet the basic requirements of cryptology, such as randomness, ergodicity, and sensitive dependence to control [1-3]. By taking advantages of these features, Çavuşoğlu et al. developed an encryption algorithm that uses chaos based S-BOX for secures and speed image encryption. Random number generator is designed with chaotic approach for this algorithm [2]. Eq. (1) shows the chaotic system used in encryption algorithm [2].

$$\begin{aligned} \dot{x} &= cy - x - bz \\ \dot{y} &= axz - xy - bx \\ \dot{z} &= dxy + b \end{aligned} \tag{1}$$

The value of system parameters and the values of initial conditions are given a = 1, b = 1, c = 2, d = –3 and $x_0 = 1$, $y_0 = -1$, $z_0 = 0.01$ respectively.

Eq. (2) shows the new chaotic system produced from the chaotic system is shown on Eq. (1) by adding a product of two parameters (zx) to the equation $\dot{z}$. Furthermore, the constant parameter 'a' can be omitted because of its value is 1 and there is no effect on the system. Then, the new chaotic attractor is shown below:

$$\begin{aligned} \dot{x} &= cy - x - bz \\ \dot{y} &= xz - xy - bx \\ \dot{z} &= dxy + b + zx \end{aligned} \tag{2}$$

Also, the value of new system parameters and the values of initial conditions are given c = 2, b = 0.3, d = -3 and $x_0 = 0.06$, $y_0 = 0.12$, $z_0 = 0.5$ respectively.

The Matlab Simulink model of the new chaotic system is given in Figure 1. Also, chaotic time series and phase portrait of the new chaotic system model system obtained by ode solver in Matlab are shown in Figure 2.

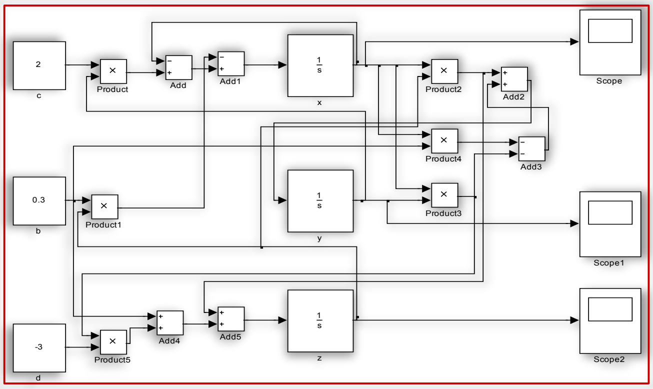
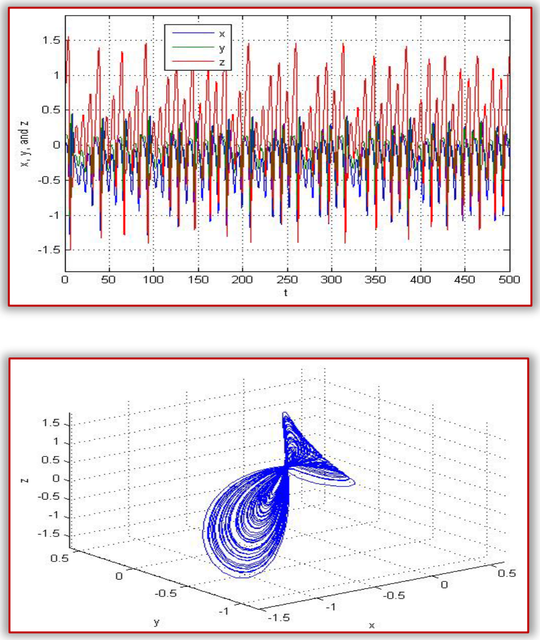


Figure 1. The Matlab-Simulink model of the system



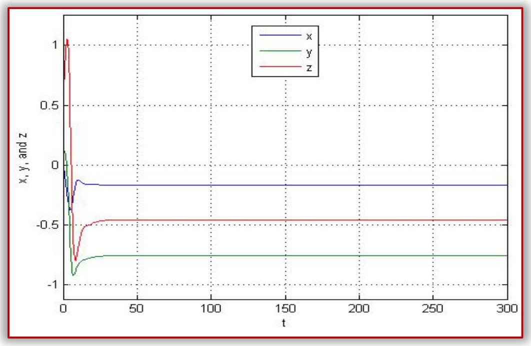
(a)

(b)

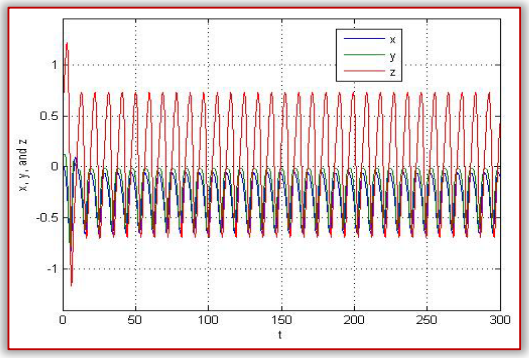
Figure 2. (a) Time series diagram and (b) phase portrait of the chaotic system

TIME SERIES ANALYSIS OF NEW CHAOTIC SYSTEM

Time series is a common type of plot used for showing the long-term behavior of ordinary differential equations of the form "$x_{n+1}=f(x_n)$, n=0,1……". This type of plot arises from a representation of the variable xn as a function of n. The horizontal axis represents number of iteration (n) and the vertical axis represents the variables (x_n) [4-7].



(a)



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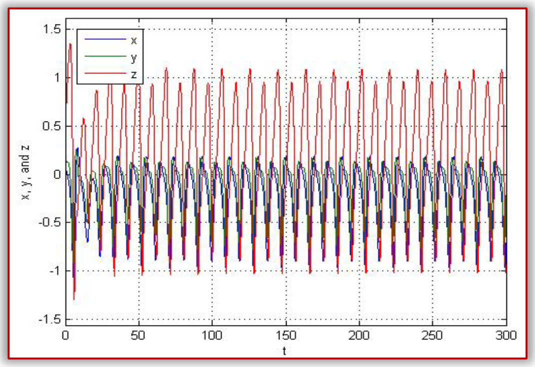
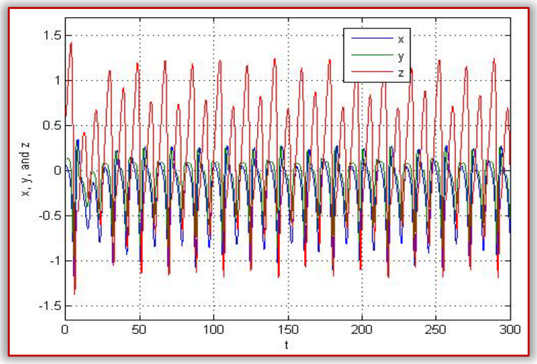


(c)



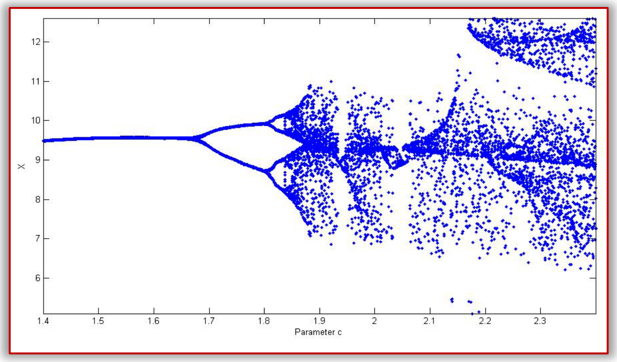
(d)

Figure 3. Time series diagram for parameter (a) c=0.4, (b) c=1.3, (c) c=1.7, and (d) c=1.84

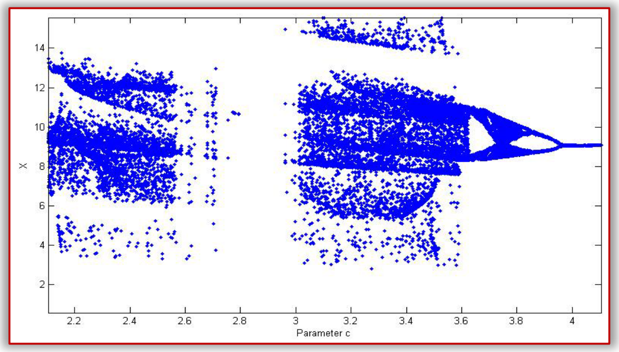
Figures 3 shows the time series plot of the attractor with respect to the changes of parameter 'c'. 300 iterations used for time parameter in these implementations and figures show the change of x, y, and z values in time. For the value of "c=0.4" system shows linear behavior and points flow to a fixed point. By increasing the value of 'c' step by step, system starts to behave chaotically. At value 1.3 the system is showing period two behavior, whereas period four behavior for c is equal to 1.7. At the Figure 3(d), the value of parameter c becomes equal to 1.84 approximately. After this value, the system is not periodic anymore and goes into chaos.

BIFURCATION DIAGRAM ANALYSIS OF NEW CHAOTIC SYSTEM

Bifurcation diagram is used to show visited or asymptotically approached values of a system as a function of a bifurcation parameter in the system [8-11]. Generally, the sequent dots represent stable values and unstable values are omitted. The bifurcation parameter is shown on the horizontal axis and the expected result of the system for that value is shown on vertical axis.



(a)

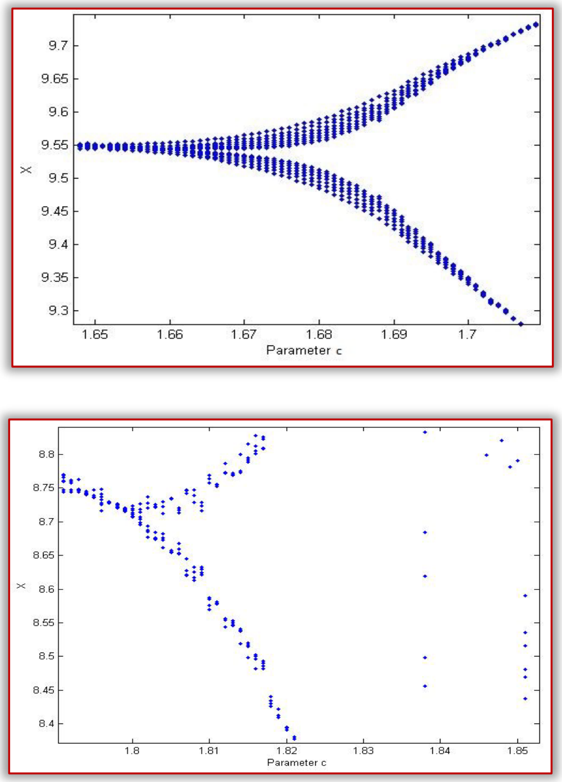


(b)

Figure 4. Bifurcation diagram of the attractor for parameter (a) 1.4<c<2.4, (b) 2.1<c<4.1.

Figure 4 shows the bifurcation diagram of the attractor for varying values of parameter c between 1.4 and 4.1.

As shown at the previous part with time series, system puts on period two behavior for 'c' values after 1.67. Then, it shows period four behavior until reaches to a value between 1.83 and 1.84. After all, the attractor enter the chaos. However, after reaching chaos the system may have some tunnels of unstable points again and again. These tunnels are named as windows of order. At the Figure 4(b), between 2.8 and 3 approximately, there occur a window of order and it can be seen again for the next values.



(a)

(b)

Figure 5. Bifurcation diagram of the attractor for parameter (a) 1.64<c<1.71, (b) 1.79<c<1.86.

LARGEST LYAPUNOV EXPONENT ANALYSIS OF NEW CHAOTIC SYSTEM

One of the methods to calculate the amount of chaos in a system is Lyapunov exponent which measure the exponential divergence of initially close state-space trajectories [12]. Diagram of largest Lyapunov exponent clues in chaotic behavior of a system. Figure 6 shows the largest Lyapunov exponent diagram of the system under the 100 initial iterations and 50000 iterations of parameter c between the values 0 and 5. E&F Chaos Program is used for this estimation [13]. Firstly, model of the system is built for the tool. Then, the parameter is chosen, minimum/maximum values of that parameter is given, and iteration numbers are selected by using "Largest Lyapunov Exponent" feature under the plot tab.

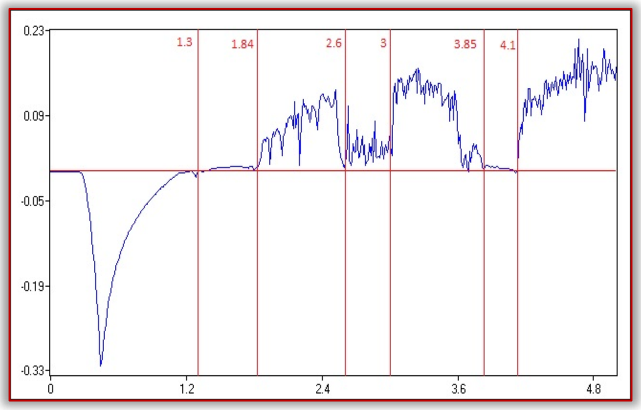


Figure 6. Largest lyapunov exponent of new chaotic system - value of c is approximately 1.3, 1.84, 2.6, 3, 3.85 and 4.1 on the red lines

When largest Lyapunov exponent graph goes into the positive area, the system starts to behave as chaotic. As mentioned at the previous part by bifurcation diagrams, the attractor behaves chaotic when the parameter c reaches approximately a value of 1.84. Therefore, largest Lyapunov exponent of the attractor is expected to go into positive values at those values shown on the Figure 6. At the first marked line, which is 1.3, the system does not go into chaos yet but possesses a periodic behavior. This results in almost linear behavior near to the zero values of largest Lyapunov exponent. However, the attractor enters chaos after the value 1.84 which is marked by the second line. Furthermore, the system is not always in chaos as mentioned at the previous parts. As seen in the bifurcation diagram on Figure 4(b), the system is in a window of order for the values between 2.8 and 3, in addition, it is also not in chaotic form for the values close to 4. As a result of that, the largest Lyapunov exponent goes down to zero or near somewhere to zero for these values as shown on the Figure 6 between 2.6 - 3 and 3.85 - 4.1 lines. Consequently, a solid relationship is seen between bifurcation diagram and largest Lyapunov exponent of the system.

FORMING MECHANISM OF NEW CHAOTIC SYSTEM

A controlled system of the new chaotic attractor is described with a newly added constant parameter 'u' in order to reveal the form mechanism of that attractor.

$$\begin{aligned} x' &= cy - x - bz \\ y' &= xz - xy - bx + u \\ z' &= dxy + b + zx \end{aligned} \tag{3}$$

Dynamical behavior of the controlled attractor (3) is studied to perform the forming mechanism. The system may show some different behaviors with varying values of parameter 'u' which is considered as the "controller" of the system [14].

Table 1 shows that the system converges to a point when the value of parameter u is large enough. By decreasing gradually, there occur limit cycle, period-doubling, and period-four behaviors. Then, the attractor becomes a complete attractor between very small values around zero. Furthermore, the system diverges from a point when the value of parameter u is small enough [14].

Table 1. Parameter range of dynamical behaviors of the controlled system 3

Parameter u	System behavior
For u ≥ 0.13 and -0.0194 > u ≥ -8.9	the system converges to a point (see Figure 7(a))
For 0.13 > u ≥ 0.103 and -8.9 > u ≥ -9.3	the system has a limit cycle (see Figure 7(b))
For 0.103 > u ≥ 0.0975	there are period-doubling behavior (see Figure 7(c))
For 0.0975 > u ≥ 0.0959	there are period-four behavior (see Figure 7(d))
For 0.0959 > u ≥ -0.0194	the system has a complete attractor. (see Figure 2)
For -9.3 > u	the system diverges from a point (see Figure 7(e))

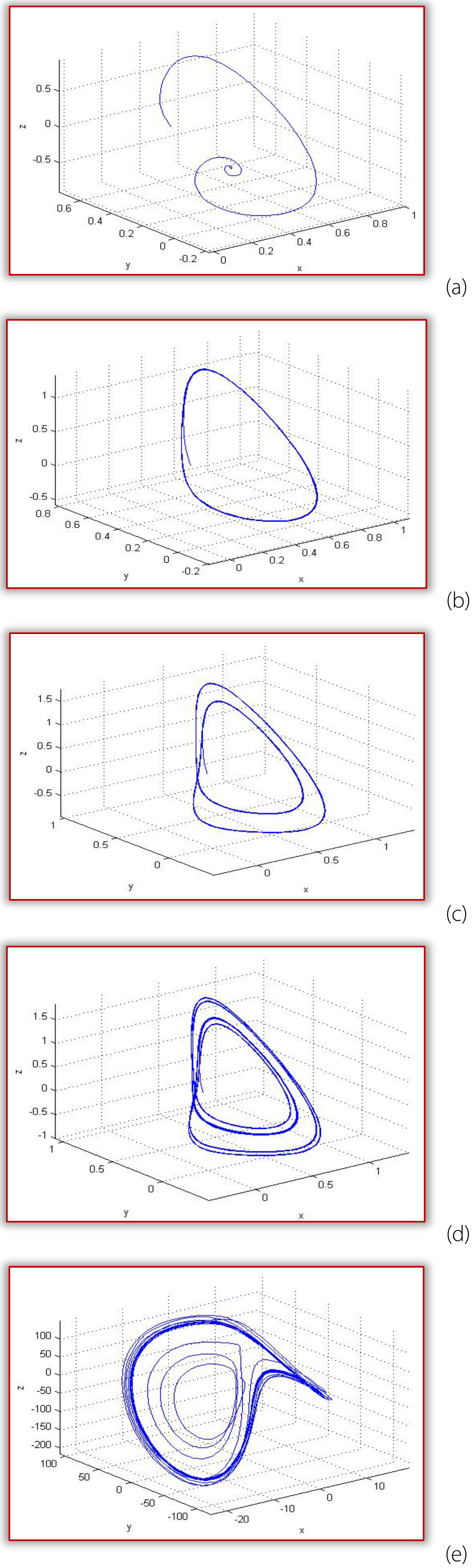


Figure 7. Phase portrait of the attractor for (a) u=0.3, (b) u=0.118, (c) u=0.099, (d) u=0.097, (e) u=-200

CONCLUSIONS

Firstly, a new chaotic attractor is found by changing an already existing one. In this study, this newfound chaotic attractor is investigated. Time series analysis, bifurcation diagram, and largest Lyapunov exponent decomposition of this new attractor are given for one of the constant parameters of it. This study has further given forming mechanism analysis of the attractor and sought the forming mechanism of it by explaining dynamical behaviors under varying values of a newly added constant parameter. Consequently, an exhaustive understanding is revealed for a new chaotic attractor through different ways.

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