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ANALYSIS OF ROTATING FUNCTIONALLY GRADED DISKS WITH ARBITRARY MATERIAL PROPERTIES

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Abstract: This paper deals with the steady-state thermoelastic problem of functionally graded rotating disks and rings with arbitrary thickness profile and material distribution within the structural component. The material properties are arbitrary functions of the radial coordinate and the temperature field. The thin disk is subjected to axisymmetric mechanical and thermal loads. The linear thermoelastic problem is solved by multilayered approach in two steps, in which the solution of the thermal part of the problem and the solution of the pure mechanical problem with constant pressure and body forces are presented. The superposition of these solutions is used to solve the original problem. The temperature field is determined by the solution of the heat conduction problem using finite difference method. The results are compared to finite element simulations.
Keywords: FGM, thermoelastic, arbitrary properties, temperature dependency

INTRODUCTION

As technology progresses at an ever increasing rate, the need for advanced materials becomes a priority in the engineering of more complex systems. This need can be seen in many fields in which engineers are exploring the applications of these new materials, such as composites or functionally graded materials. Functionally Graded Materials (FGMs) are advanced material in which the composition and structure gradually change resulting in a corresponding change in the properties of the material. In functionally graded materials the sharp interfaces between the constituent materials –which are present in composites- are eliminated. Although one of most advanced manufacturing method of FGMs known as solid freeform (SFF) method, where the components is built layer by layer. To produce bulk functionally graded components -such as disks or rings with special profile- the laser based SFF methods are utilized generally, such as 3D printing, laser cladding based method (Fig. 1), selective laser sintering and selective laser melting. SFF involves five basic steps, which are the generation of CAD data from CAD software, conversion of the CAD data to Standard Triangulation Language (STL) file, slicing the geometry (STL) into two dimensional cross section profiles, creating the component layer by layer, and the finishing operation –due to poor surface quality.

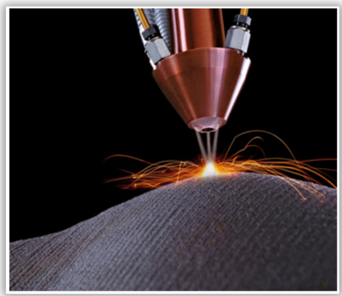

Figure 1. Additive manufacturing of FGM components [www.amcor-project.eu, (2018)]

Some textbooks such as Timoshenko and Goodier [1], Barber [2], Solecki and Conant [3], Baroumi and Ragab [4], Hetnarski and Eslami [5], Noda et. al [6] gave detailed analysis for the thermal stress problem for isotropic elastic disks with axisymmetric temperature field. Furthermore these books and papers [1-6] neglect the convective heat exchange on the lower and upper plane surfaces of the disks. Numerous papers, such as [7-11], present thermomechanical problems of functionally graded disks but the material parameters are special functions of the radial coordinate.

In a papers by Pen, X. and Li, X. [12] the thermoelastic problem of isotropic functionally graded disk with arbitrary radial non-homogeneity was considered. The numerical solution of the steady-state thermoelastic problem was reduced to a solution of a Fredholm integral equation.

Zamani N. and Rahimi [13] investigated thermal and mechanical stresses under plane stress and generalized plane strain assumptions. Concerning the stress analysis of cylindrical and spherical structural elements, Tutuncu and Temel [14] presented a novel approach to the stress analysis of pressurized FGM disks, cylindrical bodies and spheres. In these axisymmetric problems the displacements and stresses of functionally graded hollow cylinders, disks and spheres subjected to constant internal pressure were calculated using plane elasticity theory and complementary functions method.

A paper by Gönczi and Ecsedi [15] tackled the thermoelastic problems of radially graded thin disks with constant thickness and arbitrary material properties, although the body forces coming from the rotation and the temperature dependency were neglected. Multilayered approach is an effective way to approximate complex problems and solve them faster and more easily. Papers [16-18] deals with the problem of heterogeneous curved structural components, and demonstrate the efficiency and versatility of multilayered structures. Paper [19] dealt with the problem of multilayered spherical pressure vessels using the superposition of the solutions of the problem with the thermal and with the mechanical boundary conditions.

This paper tackles the steady-state problem of rotating radially graded thin disks and rings mounted on a cylindrical body, and subjected to combined axisymmetric temperature field and constant pressure. All of the material parameters are arbitrary functions of the radial coordinate r and temperature (such as E=E(r, T(r))…), while the thickness of the disk is arbitrary function of radial coordinate. The sketch of the thermoelastic problem of a functionally graded rotating disk is displayed in Figure 2. The inner radius of the disk is denoted by a, the outer radius is b and the thickness is denoted by w(r). Figure 2 shows the boundary conditions and the loading. For this problem a cylindrical coordinate system (rφz) will be used. There are thermal boundary conditions of the third-kind prescribed on the inner and outer cylindrical surfaces, furthermore there are no internal heat sources. γ_a and γ_b denote the heat transfer coefficients on the boundary surfaces (r=a and r=b, respectively). T_{ea} and T_{eb} are the environment temperatures at the inner and outer cylindrical surfaces, respectively. If $\gamma \to \infty$, then we have thermal boundary conditions of the first-kind on these surfaces.

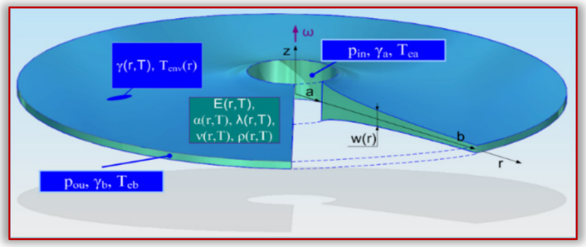

Figure 2. The disk with the loading and thermal boundary conditions

On the other two boundary surfaces the environmental temperatures are arbitrary functions of the radial coordinate. Let's assume that the heat transfer coefficient γ =γ(r, T(r)) is coordinate and temperature-dependent. The angular velocity ω is constant, p_{in} is the constant pressure exerted on the inner boundary surface, while p_{ou} is the constant outer pressure. The problem of the previously presented functionally graded disk will be solved based on the equations of the steady-state heat conduction, the field equations of the thermoelasticity and an approximate multilayered model will be used.

CALCULATION OF THE TEMPERATURE FIELD

The approximate model can be seen in Fig. 3. The number of layers is denoted by n, the layers have constant thicknesses where $w_i>0$, furthermore the material properties are discretized too. It is assumed, that the layers are perfectly bonded. Let the index i denote the quantities of the i-th layer. At first let's discretize the geometry of the disk and the thermal boundary conditions.

$$R_{mi}=\frac{R_i+R_{i+1}}{2},\lambda_i(T)=\lambda(r=R_{mi},T),$$
$$\gamma_i=\gamma(r=R_{mi},T), w_i=w(r=R_{mi}), \qquad (1)$$
$$t_{env,i}=T_{env}(r=R_{mi}),\ i=1,...,n,$$

where λ denotes the thermal conductivity, t denotes the specific values of the temperature function at certain radial coordinate values.

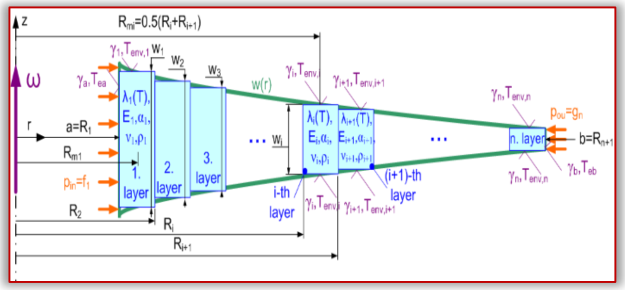

Figure 3. The approximate model of the functionally graded disk

For this case the nonlinear differential equation for the temperature field of the i-th layer ($T_i(r)$) has the following form [20]:

$$\frac{1}{r}\frac{d}{dr}\left(r\lambda_i\frac{dT_i}{dr}\right)-\frac{2\gamma_i}{w_i}\left(T_i-t_{env,i}\right)=0. \qquad (2)$$

Finding the closed form analytical solution for differential Eq. (2) is very hard, therefore a numerical method will be utilized to solve it. The points of the temperature field will be calculated with finite difference method. The nonlinear system of equations (with m points in each layer, the number of layers is n) for the whole model can be expressed as:

$$\begin{aligned}0=&\frac{\lambda_i\left(T=t_{k+(i-1)m}\right)}{a+(i-1)d_m+k\Delta r}\frac{t_{k+(i-1)m}-t_{k+(i-1)m-1}}{\Delta r}+\\&+\frac{t_{k+(i-1)m}-t_{k+(i-1)m-1}}{\Delta r}\frac{d\lambda_i\left(T=t_{k+(i-1)m}\right)}{dr}+\\&+\frac{t_{k+(i-1)m+1}-2t_{k+(i-1)m}+t_{k+(i-1)m-1}}{\Delta r^2}\lambda_i\left(T=t_{k+(i-1)m}\right)-\\&-\frac{2\gamma_i(T=t_{k+(i-1)m})}{w_i}\left(t_{k+(i-1)m}-t_{env,i}\right),\end{aligned} \qquad (3)$$

$$\text{where } d_m=\frac{b-a}{n},\ \Delta r=\frac{b-a}{nm},$$
$$i=1,...,n,\ \text{while}\ k=1,...,m-1.$$

In many cases the effective material properties can be expressed as a nonlinear functions of the temperature [21]:

$$\mathbf{Mp(T)=P_0(P_{-1}t^{-1}+1+P_1t+P_2t^2+P_3t^3)}. \qquad (4)$$

In Eq. (4) Mp(T) denotes the function of the considered effective material property, P_0, P_{-1}, P_1, P_2 and P_3 are material dependent coefficients of absolute temperature t in [*K*]. Using these results we can present functions for the temperature- and position-dependent functionally graded material parameters [21]:

$$Mp_f(r,T)=\left[Mp_1(T)-Mp_2(T)\right]\left[G(r)\right]^m+Mp_2(T)$$
$$\text{e.g. for disks or spheres: } G(r)=\frac{r-a}{b-a}, \qquad (5)$$

furthermore, indices 1 and 2 denote the constituent materials – in many cases metal and ceramic components. If the thermal conductivity has the form of Eqs. (4,5) then we write for the previous system of nonlinear Eqs. (3):

$$\lambda_i\left(T=t_{k+(i-1)m}\right)=K_{0,i}+K_{-1,i}t^{-1}_{k+(i-1)m}+ +K_{1,i}t_{k+(i-1)m}+K_{2,i}t^{2}_{k+(i-1)m}+K_{3,i}t^{3}_{k+(i-1)m}, \tag{6}$$

$$\frac{d\lambda_i\left(T=t_{k+(i-1)m}\right)}{dr}=(-K_{-1,i}t^{-2}_{k+(i-1)m}+2K_{2,i}t_{k+(i-1)m}+ +K_{1,i}+3K_{3,i}t^{2}_{k+(i-1)m})\frac{t_{k+(i-1)m}-t_{k+(i-1)m-1}}{\Delta r}, \tag{7}$$

where

$$\begin{aligned} K_{-1,i}&=\left(P^1_{-1}\lambda^1_0-P^2_{-1}\lambda^2_0\right)\times G(r=R_{mi})+P^2_{-1}\lambda^2_0,\\ K_{0,i}&=\left(\lambda^1_0-\lambda^2_0\right)\times G(r=R_{mi})+\lambda^2_0,\\ K_{1,i}&=\left(P^1_{1}\lambda^1_0-P^2_{1}\lambda^2_0\right)\times G(r=R_{mi})+P^2_{1}\lambda^2_0,\\ K_{2,i}&=\left(P^1_{2}\lambda^1_0-P^2_{2}\lambda^2_0\right)\times G(r=R_{mi})+P^2_{2}\lambda^2_0,\\ K_{3,i}&=\left(P^1_{3}\lambda^1_0-P^2_{3}\lambda^2_0\right)\times G(r=R_{mi})+P^2_{3}\lambda^2_0, \end{aligned} \tag{8}$$

where λ^j_0 (j=1,2: number of the constituent material) are material constants. We assume that the surface temperatures of the adjacent layers are equal and the radial heatflow q is constant.

$$-w_i\lambda_i\left(T=t_{(i-1)m}\right)\frac{t_{(i-1)m+1}-t_{(i-1)m}}{\Delta r}= =-w_{i-1}\lambda_{i-1}\left(T=t_{(i-1)m}\right)\frac{t_{(i-1)m}-t_{(i-1)m-1}}{\Delta r}, i=2,...,n. \tag{9}$$

From the third-kind thermal boundary conditions it follows that

$$\begin{aligned} &\frac{t_2-t_1}{\Delta r}-(t_1-t_{env,a})w_1(T=t_1)=0,\\ &\frac{t_{nm}-t_{nm-1}}{\Delta r}+(t_{nm}-t_{env,b})w_n(T=t_{nm})=0. \end{aligned} \tag{10}$$

The points of the temperature field can be calculated from the nonlinear system of equations (3), (9) and (10). Then a polynomial curve can be fitted to these calculated values (via least squares method), for power-law distributions and for smaller power index values (m<7) we can use

$$T_{appr}=J_6r^6+J_5r^5+J_4r^4+J_3r^3+J_2r^2+ +J_1r+J_0+J_{-1}r^{-1}+J_{-2}r^{-2}+J_{-3}r^{-3}. \tag{11}$$

THE SOLUTION OF THE THERMOELASTIC PROBLEM

In our case the time dependency of the problem is neglected, and the thermoelastic problem is uncoupled which means that previously determined temperature field is an input function for the field equations of the boundary value problem of linear elasticity and $Mp_f(r,T(r))\rightarrow Mp_f(r)$. The thermoelastic problem will be divided into two parts, then the principle of superposition will be used to solve it.

Using the approximate temperature field, the material parameters for each layer can be discretized as

$$\begin{aligned} E_i&=E(r=R_{mi},T=t_{mi}),\\ \nu_i&=\nu(r=R_{mi},T=t_{mi}),\\ \alpha_i&=\alpha(r=R_{mi},T=t_{mi}),\\ \rho_i&=\rho(r=R_{mi},T=t_{mi}),\\ &i=1,...,n, \end{aligned} \tag{12}$$

where E is the modulus of elasticity, ν is Poisson's ratio, α is the coefficient of linear thermal expansion and ρ denotes the density of the material. At first we consider the case when the i-th layer is under thermal loading and has a steady-state temperature field, furthermore the boundary surfaces of the layers are assumed to be traction free. The $u_i^T(r)$ thermal radial displacement function and the $\sigma_{rr,i}^T(r)$, $\sigma_{\varphi\varphi,i}^T(r)$ thermal stresses can be determined as [22]:

$$u^T_i(r)=\frac{1+\nu_i}{r}\alpha_i\int_{R_i}^{r}rT_i(r)dr+ +\frac{(1+\nu_i)R_i^2+(1-\nu_i)r^2}{r(R_{i+1}^2-R_i^2)}\alpha_i\int_{R_i}^{R_{i+1}}rT_i(r)dr, \tag{13}$$

$$\sigma^T_{rr,i}(r)=\frac{\alpha_iE_i}{R_{i+1}^2-R_i^2}\left(1-\frac{R_i^2}{r^2}\right)\int_{R_i}^{R_{i+1}}rT_i(r)dr- -\frac{\alpha_iE_i}{r^2}\int_{R_i}^{r}rT_i(r)dr, \tag{14}$$

$$\sigma^T_{\varphi\varphi,i}(r)=E_i\alpha_i\left[\frac{1}{r^2}\int_{R_i}^{r}rT_i(r)dr-T_i(r)+ \frac{1}{R_{i+1}^2-R_i^2}\left(\frac{R_i^2}{r^2}+1\right)\int_{R_i}^{R_{i+1}}rT_i(r)dr\right], i=1,...,n. \tag{15}$$

The sketch of a layer is illustrated in Fig. 4.

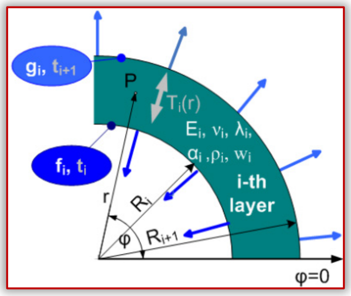

Figure 4. The sketch of a quarter of the i-th layer

We will assume that there are constant mechanical loading $f_i=\sigma^M_{rr,i}(R_i)$ and $g_i=\sigma^M_{rr,i}(R_{i+1})$ on the inner and outer cylindrical boundary surfaces of the *i*-th layer. The differential equation of the radial displacement field can be derived from the basic equation of linear elasticity for the plane stress state of thin disks (equilibrium equation, kinematic equations and stress-strain relations):

$$\frac{d\sigma_{rr}}{dr}+\frac{\sigma_{rr}-\sigma_{\varphi\varphi}}{r}+\omega^2\rho r=0, \tag{16}$$

$$\begin{aligned} &\varepsilon_{rr}=\frac{du}{dr},\quad \varepsilon_{\varphi\varphi}=\frac{u}{r},\\ &\sigma_{rr}=\frac{E}{1-\nu^2}\left[\varepsilon_{rr}+\nu\varepsilon_{\varphi\varphi}-\alpha(1+\nu)T\right],\\ &\sigma_{\varphi\varphi}=\frac{E}{1-\nu^2}\left[\nu\varepsilon_{rr}+\varepsilon_{\varphi\varphi}-\alpha(1+\nu)T\right]. \end{aligned} \tag{17}$$

$$\begin{aligned} &\frac{d^2u_i^M(r)}{dr^2}+\frac{du_i^M(r)}{dr}\frac{1}{r}-\frac{u_i^M(r)}{r^2}+K_ir=0,\\ &K_i=\frac{(1-\nu_i^2)\rho_i\omega^2}{E_i}. \end{aligned} \tag{18}$$

After solving Eq. (18) we get the following expressions for the displacement field and the normal stresses:

$$u_i^M(r)=C_i r+\frac{B_i}{r}-\frac{K_i}{8}r^3, \quad (19)$$

$$\sigma_{rr,i}^M(r)=\frac{E_iC_i}{1-\nu_i}-\frac{E_iB_i}{1+\nu_i}\frac{1}{r^2}-A_ir^2,$$

$$A_i=\frac{E_i(3+\nu_i)K_i}{8(1-\nu_i^2)}, \quad (20)$$

$$\sigma_{JJ,i}^M(r)=\sigma_{\varphi\varphi,i}^M(r)=\frac{E_iC_i}{1-\nu_i}+\frac{E_iB_i}{1+\nu_i}\frac{1}{r^2}-A_ir^2, \quad (21)$$

$$i=1,...,n.$$

Using the equations of the boundary conditions, the unknown parameters B_i and C_i can be determined:

$$B_i=\frac{(1+n_i)R_{i+1}^2R_i^2(A_i(R_i^2-R_{i+1}^2)+f_i-g_i)}{E_i(R_i^2-R_{i+1}^2)}, \quad (22)$$

$$C_i=\frac{(1-\nu_i)(A_i(R_{i+1}^4-R_i^4)+R_{i+1}^2g_i-R_i^2f_i)}{E_i\left(R_i^2-R_{i+1}^2\right)}. \quad (23)$$

The superposition principle is utilized for this problem, because both the previously used field equations and the boundary conditions are linear. This means that we can add the stresses and displacements caused by the mechanical loading (8-13) to the thermal stresses and displacements (13-15) in order to solve this problem. For the computation of the combined loads the following equations are used:

$$u_i(r)=u_i^T(r)+u_i^M(r), \quad (24)$$

$$\sigma_{rr,i}(r)=\sigma_{rr,i}^T(r)+\sigma_{rr,i}^M(r), \quad (25)$$

$$\sigma_{\varphi\varphi,i}(r)=\sigma_{\varphi\varphi,i}^T(r)+\sigma_{\varphi\varphi,i}^M(r),\ i=1,\dots,n. \quad (26)$$

The unknown parameters f_i (i=2...n) and g_i (i=1...n-1) in the equations (19-23) can be calculated from the following system of equations

$$u_i(R_{i+1})=u_{i+1}(R_{i+1}),\ i=1,\dots,n-1, \quad (27)$$

which ensure the continuity of the radial displacement field furthermore f_1 and g_n are given.

$$\sigma_{rr,1}(R_1)=f_1=-p_{in}, \quad (28)$$

$$\sigma_{rr,n}(R_{n+1})=g_n=-p_{ou}.$$

The system of equations (27) has the following form with 2(n-1) equations:

$$u_{i+1}^T(R_{i+1})-u_i^T(R_{i+1})=$$

$$\left[\frac{(1-\nu_i)(A_i(R_{i+1}^4-R_i^4)+R_{i+1}^2g_i-R_i^2f_i)}{E_i\left(R_i^2-R_{i+1}^2\right)}R_{i+1}+\right.$$

$$\left.+\frac{(1+\nu_i)R_{i+1}^2R_i^2(A_i(R_i^2-R_{i+1}^2)+f_i-g_i)}{E_i(R_i^2-R_{i+1}^2)R_{i+1}}-\frac{K_i}{8}R_{i+1}^3\right]- \quad (29)$$

$$-\left[\frac{(1-\nu_{i+1})(A_{i+1}(R_{i+2}^4-R_{i+1}^4)+R_{i+2}^2g_{i+1}-R_{i+1}^2f_{i+1})}{E_{i+1}\left(R_{i+1}^2-R_{i+2}^2\right)}R_{i+1}+\right.$$

$$+\frac{(1+\nu_{i+1})R_{i+1}^2R_{i+2}^2(A_{i+1}(R_{i+1}^2-R_{i+2}^2)+f_{i+1}-g_{i+1})}{E_{i+1}(R_{i+1}^2-R_{i+2}^2)R_{i+1}}-$$

$$\left.-\frac{K_{i+1}}{8}R_{i+1}^3\right],\quad g_i=\frac{w_{i+1}}{w_i}f_{i+1}.$$

Using the previously determined parameters f_i, g_i and equations (24-26) the radial displacements and the normal stresses of the multilayered body can be calculated. Due to the multilayered model the curve of the tangential normal stress function may contain significant steps, but in certain cases there are certain point where the stress values have good accuracy. For example the middle points of the layers of a disk with constant thickness (w=const.) have the least errors, thus fitting an approximate curve is recommended.

NUMERICAL EXAMPLES

In this section a numerical example is presented for rotating radially graded disks with a prescribed $w(r)$ thickness and temperature-dependent material properties. The results of the presented methods are compared to results obtained by finite element simulation in Abaqus. The following numerical data will be used for the computations:

a=0.1m, b=0.3m, w=0.0115-0.025r [m],
$T_{env}(r)=398(r-0.09)^{0.01}a^{-0.01}-t_{ref}$ [K], [K],
T_{in}=100K, T_{ou}=400K, t_{ref}=273K, m=3 and for λ
and γ: m_1=2.3, p_{in}=60MPa, p_{ou}=0MPa, ω=400 rad/s.

Table 1 contains the material properties of the constituent materials based on Eqs. (4) and (5). Figure 5 shows that the temperature field of thin disks depends only on the radial coordinate by these symmetric boundary conditions.

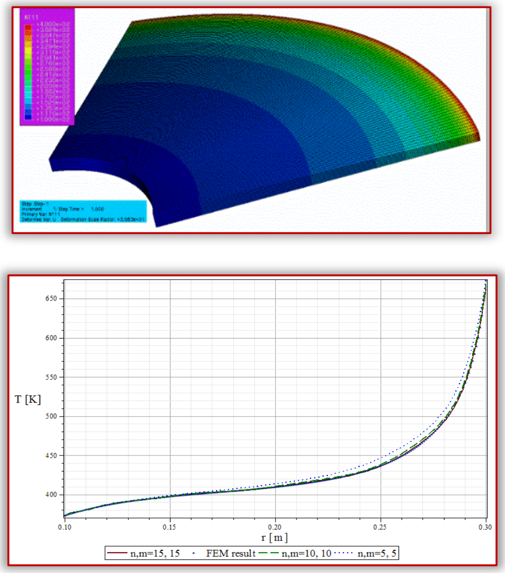

Figure 5. The finite element model with the absolute temperature field and the graphs

Table 1. Material properties of the FGM

Material Property (MP_f)	Material (1)			
	P_{m0}	$P_{m1}(10^{-3})$	$P_{m2}(10^{-7})$	$P_{m3}(10^{-10})$
λ(W/mK)	15.39	-2.364	20.92	-7.223
γ(W/m²K)	10	0	0	0
ρ(kg/m³)	7200	0.3079	-6.53	0
α (1/K)	$12.33\cdot10^{-6}$	0.8086	0	0
E (Pa)	$2.01\cdot10^{11}$	0.3079	-6.53	0
ν (-)	0.326	-0.1	0.38	0
(MP_f)	Material (2)			
	P_{c0}	$P_{c1}(10^{-3})$	$P_{c2}(10^{-7})$	$P_{c3}(10^{-11})$
λ(W/mK)	1.7	-0.1276	0.06648	-1
γ(W/m²K)	2	0	0	0
ρ(kg/m³)	10^4	-0.307	2.16	-8.946
α (1/K)	$3.87\cdot10^{-6}$	0.909	0	0
E (Pa)	$3.484\cdot10^{11}$	-0.31	2.16	-8.94
ν (-)	0.24	0	0	0

The results of the displacements and normal stresses are in good agreement as it can be seen in Figs. 6-8. The approximation of the normal stresses can improve the accuracy of the multilayered method. For the finite element calculations Abaqus CAE FE software was used. The two dimensional axisymmetric model can be seen in Fig. 6, the method of modelling is similar to the one presented in paper [23].

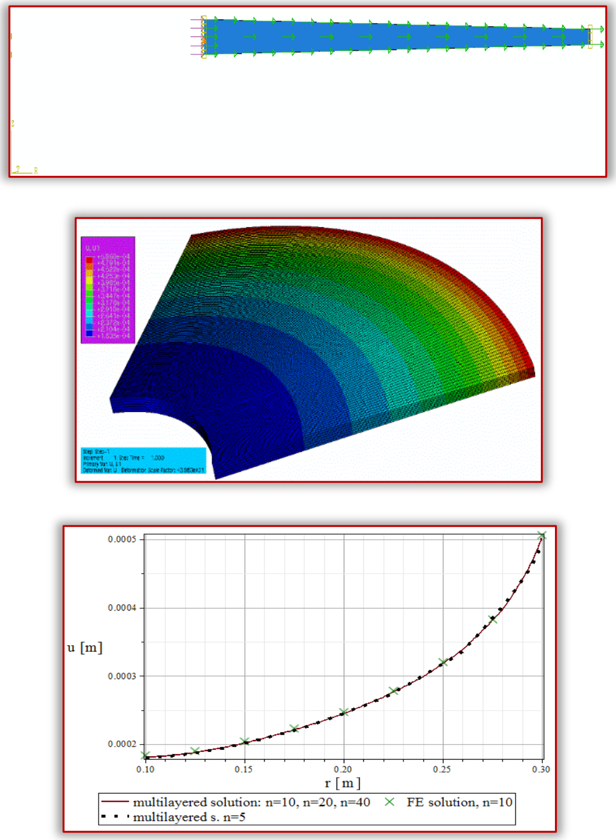

Figure 6. The finite element model of the disk with the radial displacement and the graphs of the different solutions

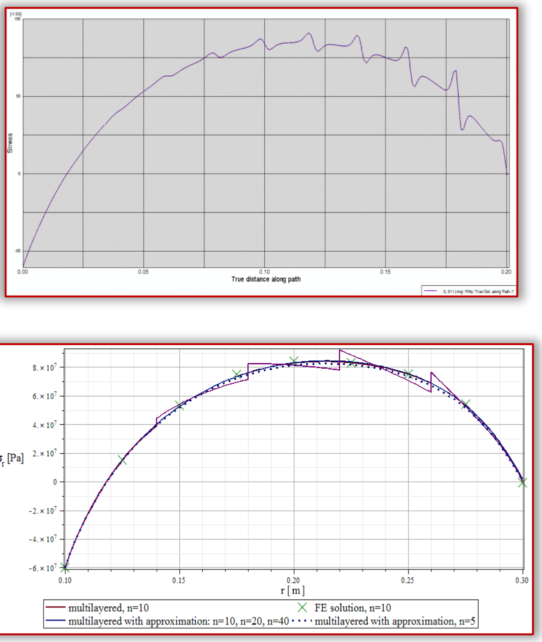

Figure 7. The radial normal stresses within the radially graded disk

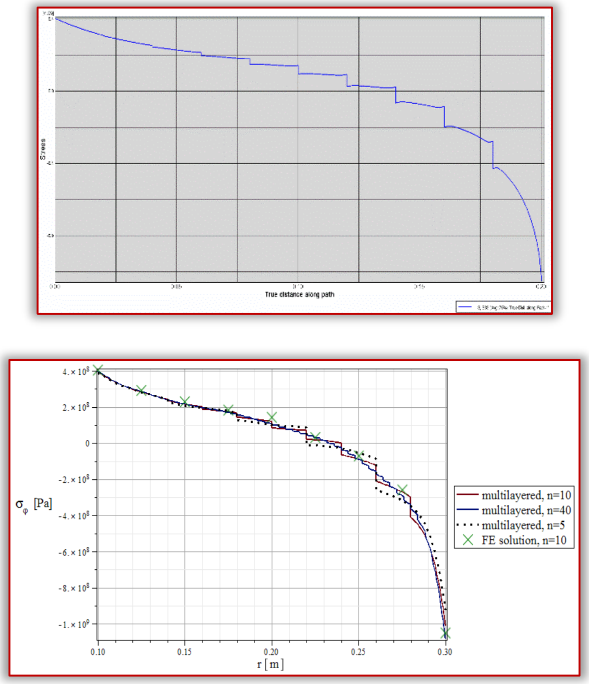

Figure 8. The graphs of the tangential normal stresses

For disks with complicated thickness function, we can use different point –where the relative errors are small- to fit approximate curves to them. For example in this numerical problem, the average values of the tangential stresses at the boundary surfaces of the layers have decent accuracy.

CONCLUSIONS

A steady-state thermoelastic problem of radially graded thin disks subjected to combined mechanical and thermal loads was solved based on a multilayered approach and the principle of superposition. The thickness of the disk was an arbitrary function of the radial coordinate, while the material properties were arbitrary functions of the radial coordinate and the temperature field. The temperature field was calculated using the method of finite differences, and was only the function of the radial coordinate.

The results were compared to finite element calculations and they were in good agreement. One of the advantages of the method presented in this paper -over FEM- is the speed of the calculation for the elasticity problem and the accuracy of the method can be improved with properly chosen approximate function. Furthermore finite element software are expensive, out of reach for small- and medium-sized companies which makes other numerical solutions more desirable.

References

[1] Timoshenko S., Goodier I. N.: Theory of Elasticity, McGraw-Hill, New York, 1951.
[2] Barber I. N.: Intermediate Mechanics of Materials, 2nd edition, Springer, New York, 2011.
[3] Solecki R., Conant R, I.: Advanced Mechanics of Materials, Oxford University Press, Oxford, 2003.
[4] Ragab A. R., Bayoumi S. E.: Engineering Solid Mechanics (Fundamentals and Applications), CRC Press, London, 1999.
[5] Hetnarski, R. B., Eslami, M. R.: Thermal Sresses – Advaned Theory and Applications, Springer, New York, 2010.

[6] Noda N., Hetnarski R. B. and Tanigawa Y.: Thermal Stresses. Lastran Corporation, Rochester, New York, 2000.
[7] Loghman A., Ghorbanpour A. A., Shajari R. A., Amir s.: Time-dependent thermoelastic creep analysis of rotating disk made of Al–SiC composite. Archive of Applied Mechanics, Springer, 81 (12), 1853-1864, 2011.
[8] Afsar A. M., Go J.: Finite element analysis of thermoelastic field in a rotating FGM circular disk. Applied Mathematical Modelling, Elsevier, 34(11)., 3309-3320, 2010.
[9] Hassani A., Hojjati M. H., Mahdavi E., Alashti R. A., Farrahi G.: Thermo-mechanical analysis of rotating disks with non-uniform thickness and material properties. International Journal of Pressure Vessels and Piping, Elsevier, 98., 95-101, 2012.
[10] Dhamirchelli M., Azadi M.: Temperature and thickness effects on thermal and mechanical stresses of rotating FG-disks. Journal of Mechanical Science and Technology, Springer, pp. 11-23., 2011.
[11] Vivio F., Vullo V., Cifani P.: Theoretical stress analysis of rotating hyperbolic disk without singularities subjected to thermal load. Journal of Thermal Stresses. 37, 117–136., 2014.
[12] Pen X., Li X.: Thermoelastic analysis of functionally graded annulus with arbitrary gradient, Applied Mathematics and Mechanics (English Edition), 1211-1220, 2009.
[13] Zamani N. M., Rahimi G.H.: Deformations and stresses in rotating FGM pressurized thick hollow cylinder under thermal load. Sci. Res. Essay, 4(3), 131–140, 2009.
[14] Tutuncu N., Temel B.: A novel approach to stress analysis of pressurized FGM cylinders, disks and spheres. Compos. Struct., 91, 385–390, 2009.
[15] Gönczi D., Ecsedi I.: Thermoelastic analysis of functionally graded hollow circular disk. Achieve of Mechanical Engineering, Vol 62. (1), 5-18, 2015.
[16] Kiss L.P., Szeidl Gy.: Vibrations of pinned-fixed heterogeneous circular beams pre-loaded by a vertical force at the crown point, Journal of Sound and Vibration, Vol 393.C, 92-113, 2017.
[17] Kiss L.P., Szeidl Gy.: In-plane stability of fixed-fixed heterogeneous curved beams under a concentrated radial load at the crown point, Technische Mechanik, Vol 35 (1), 31-48, 2015.
[18] Lengyel Á., Ecsedi I.: Analysis of bimetallic beam with weak shear connection. Acta Technica Corviniensis – Bulletin of Engineering, Vol 9. (3), 85-90, 2016.
[19] Gönczi D.: The determination of displacement field and normal stresses in multilayered spherical bodies. XXVIII. MicroCAD International Scientific Conference, Miskolci Egyetem (ME), Paper D35, 2014.
[20] Carslaw H. S., Jaeger I. C.:Conduction of Heat in solids, Clarendon Press, Oxford, 1959.
[21] Shen H-S. Functionally Graded Materials: Nonlinear Analysis of Plates and Shells. CRC Press, London, 2009.
[22] Boley B. A., Weiner J. H.: Theory of Thermal Stresses, John Wiley & Sons Inc., New York, 1960.
[23] Gönczi D., Ecsedi I.: Finite element modelling of functionally graded spherical pressure vessels. Annals of Faculty of Engineering Hunedoara- Internationl Journal of Engineering, Vol 12. (2), 179-184, 2014.

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