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CRACK PROPAGATION ANALYSIS OF CYCLICALLY LOADED STRUCTURAL COMPONENTS

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Abstract: The present paper proposes a computational model for the failure analysis of finite plate with quarter-elliptical corner crack located at a hole. Such a theoretical investigation takes into account the stress intensity factor calculation and fatigue life estimation. Fracture mechanics based analytical approach is employed to analyze the stress-intensity behaviour, whereas the stress-ratio dependence crack growth model is applied for assessing fatigue life to failure. The crack growth estimations show a good correlation with experimental data.

Keywords: fatigue, quarter-elliptical crack, stress-intensity behaviour, residual life calculation

INTRODUCTION

The integrity of engineering structures may be often threatened by fatigue crack-like flaws, i.e. damages during service operations. Therefore, significantly important aspect is to estimate the failure strength of components by taking into account reliable fracture mechanics based computational models.

Under cyclic loading, the crack propagation behaviour can be theoretically examined either as through-the-thickness crack situations or surface problems (quarter-elliptical corner crack, semi-elliptical crack and embedded-elliptical crack) through the stress analysis and residual life assessment by means of analytical methods and numerical approaches. Thus, Elber [1] suggested the crack closure concept, whereas Walker [2] and later Kujawski [3] proposed the two-parameter driving force model for the fatigue life estimations. Glinka et al. [4] found that the damage accumulation ahead of the crack tip can be employed to describe the crack growth rate. Furthermore, different fracture mechanics based numerical models such as: the finite element alternating method [5], the boundery integral equation method [6], the finite element method with singularity elements [7, 8] can be applied to assess the stress-intensity behaviour under fatigue conditions.

In the present paper, a mathematical model is developed for fatigue failure analysis of quarter-elliptical corner crack emanating from a hole. The proposed model considers the stress-intensity analysis and fatigue life estimation for depth and surface crack length directions. Relevant experimental crack growth data are employed to verify the predictive capability of failure assessments.

RESIDUAL LIFE ESTIMATION UNDER CYCLIC LOADING

The complex interaction of applied load and environment can often endanger the service life of engineering structures due to appearance of fatigue failure. In order to ensure safe exploitation, the reliable computational models for the fatigue analysis have to be developed by taking into account appropriate fracture mechanics principles. Thus, the propagation of quarter-elliptical corner crack emanating from a hole is herein theoretically examined through the crack growth law proposed by Walker [2[in depth and surface crack directions, as follows:

$$\frac{da}{dN}=\frac{C_A\left(\Delta K_{IA}(a,b)\right)^{m_A}}{(1-R)^2}$$

and

$$\frac{db}{dN}=\frac{C_B\left(\Delta K_{IB}(a,b)\right)^{m_B}}{(1-R)^2} \qquad \text{(1a-b)}$$

where a, b and ΔK_{IA}, ΔK_{IB} are the crack length and the stress intensity factor in depth and surface crack directions, respectively, R is the stress ratio and C_A, C_B, m_A, m_B denote material constants experimentally obtained.

In the residual strength assessment, the final number of loading cycles is computed by integrating the relationship related to the crack growth rate for depth and surface crack direction, respectively i.e.

$$N=\int_{a_0}^{a_f}\frac{(1-R)^2\,da}{C_A\left(\Delta K_{IA}(a,b)\right)^{m_A}}$$

and

$$N=\int_{b_0}^{b_f}\frac{(1-R)^2\,db}{C_B\left(\Delta K_{IB}(a,b)\right)^{m_B}} \qquad \text{(2a-b)}$$

where a_0, b_0 and a_f, b_f are initial and final crack lengths in depth and surface direction, respectively.

Since the relationships for the crack growth rate and the stress intensity factor are the complex-valued functions, the numerical integration based on the Euler's algorithm is taken into account. Thus, according to the software program here examined, the fatigue life to failure is estimated by applying Eqs. (2a)-(2b) step-by-step for appropriate crack growth increments.

STRESS-INTENSITY ANALYSIS OF QUATER-ELLIPTICAL CORNER CRACK

From the fracture mechanics theoretical point of view, the crack propagation under cyclic loading may be examined if the geometry of structural component together with parameters related to material and external loading are involved through the stress intensity factor. In the present study, the stress-intensity behaviour of the quarter-elliptical corner crack emanating from a hole (Fig. 1) is analyzed by taking into account the following relationship [9]:

$$\Delta K_I = \Delta S \sqrt{\frac{\pi a}{Q}} M_e f_1 \sqrt{\frac{1}{\cos\left(\frac{\pi D}{2w}\right)}} g_\phi \qquad (3)$$

where ΔK_I is the stress intensity factor range, ΔS denotes applied stress/load, D and w are diameter and width of the plate, respectively. Further, Q, M_e, f_1 and g_ϕ represent fracture mechanics based functions related to crack configuration and loading, expressed in the following way [9]:

$$Q = 1 + 1.47\left(\frac{a}{b}\right)^{1.64}, \left(\frac{a}{b} \le 1.0\right) \qquad (4)$$

$$M_e = \left(M_1 + \left(\sqrt{Q\frac{b}{a}} - M_1\right)\left(\frac{a}{t}\right)^p\right) f_{w1} \qquad (5)$$

$$M_1 = 1.2 - 0.1\frac{a}{b}, \left(0.02 \le \frac{a}{b} \le 1.0\right) \qquad (6)$$

$$p = 2 + 8\left(\frac{a}{b}\right)^2 \qquad (7)$$

$$f_1 = 0.707 - 0.18\lambda + 6.55\lambda^2 - 10.54\lambda^3 + 6.85\lambda^4 \qquad (8)$$

$$\lambda = \frac{1}{1 + \frac{2b}{D}\cos(0.85\phi)} \qquad (9)$$

$$f_{w1} = \sqrt{\frac{1}{\cos\left(\frac{\pi}{2}\frac{D+b}{w-b}\right)}} \qquad (10)$$

$$g_\phi = 1 + \left(0.1 + 0.35\left(\frac{a}{t}\right)^2\right)(1 - \sin\phi) \qquad (11)$$

where t is the thickness of the plate and ϕ denotes parametric angle of ellipse.

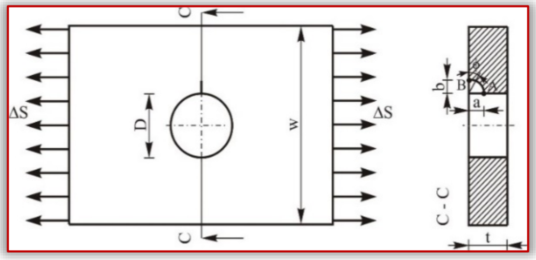


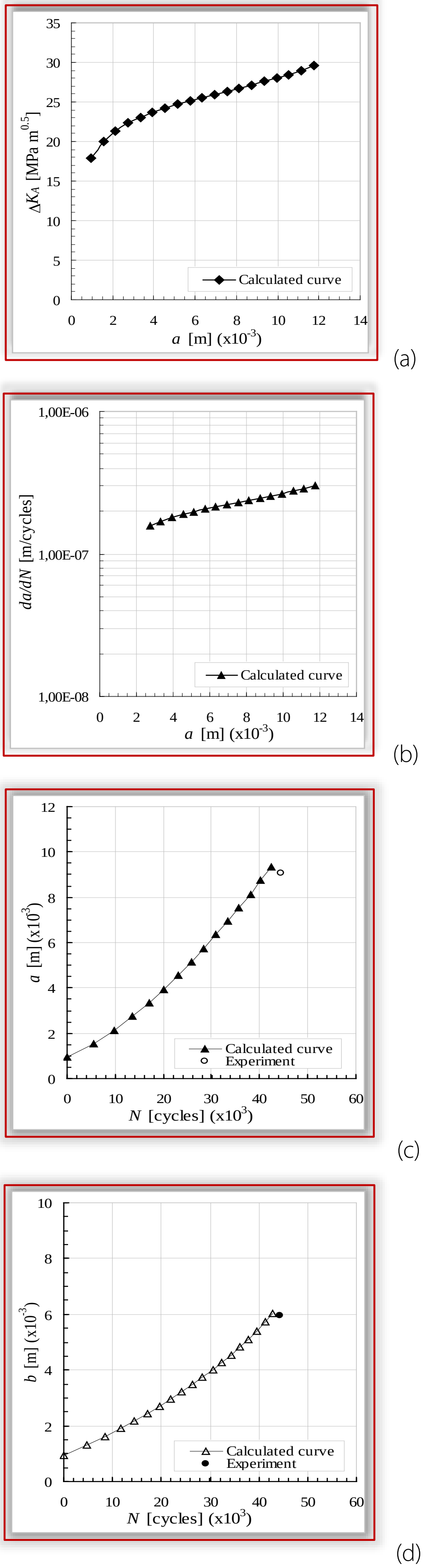
Figure 1. Geometry of the plate with quarter-elliptical corner crack emanating from a hole (a - depth direction, b – surface direction).

NUMERICAL APPLICATIONS

In the present Section, the failure behaviour of quarter-elliptical corner crack emanating from a hole is theoretically examined through the proposed computational model. Such fatigue crack growth investigation takes into account the stress intensity factor calculation and the residual life estimation.

The crack growth estimation under cyclic loading

The fatigue strength of the plate with quarter-elliptical crack emanating from a hole (Fig. 1) is tackled through the residual life calculation.



(a)

(b)

(c)

(d)

Figure 2. The fatigue failure analysis of the plate: (a) ΔK_A-a, (b) da/dN-a, (c) a-N, (d) b-N.

The crack growth process is here analyzed under cyclic axial loading wirh constant amplitude (S_{max} = 155.69 MPa, R = 0.05). The plate (w = 73.66 mm, t = 12.7 mm, D = 12.7 mm, $a_0 = b_0$ = 0.9398 mm) is made of Ti-6Al-4V Alloy and the following material parameters are assumed: C_A=1.12 10^{-10}, C_B= 1.03 10^{-10} and m_A=m_B= 2.3.

The fatigue behaviour of quarter-elliptical corner crack is examined by employing Eqs. (3)-(11) for the stress intensity factor evaluation, then the residual life is assessed through Eqs. (2a)-(2b). The calculations related to the stress intensity factor and the crack growth rate are presented in Figs. 2a and b, respectively. Further, the number of loading cycles to failure, as a function of crack length for depth and surface direction, are shown in Figs. 2c and d, respectively.

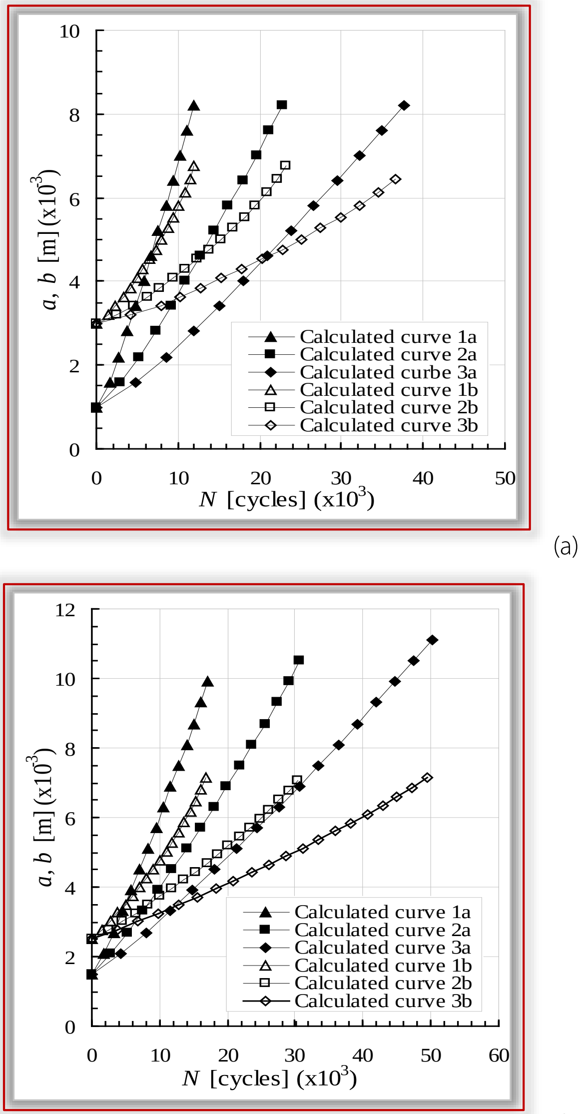
Then, to assess the predictive capability of developed computational model for the fatigue analysis of quarter-elliptical corner crack located at a hole, the residual life calculations are compared with experimental observations, as shown in Fig. 2. Hence, such comparisons indicate that proposed model here examined enables the reliable fatigue strength estimation for both depth and surface crack length directions.

The effect of thickness and width on the fatigue failure behaviour

The crack growth analysis, presented in this Section, examines the effect of thickness and width of the plate on the fatigue life to failure. In such investigation, the plate with quarter-elliptical crack, made of Ti-6Al-4V Alloy, has the following geometry and loading parameters for two cases tackled here: (a) a_0=1.5 mm, b_0= 2.5 mm, D = 15 mm, w = 65 mm, P_{max}= 130000 N, R = 0.1, t = (12, 15, 18) mm and (b) a_0=1.0 mm, b_0=3.0 mm, D = 10 mm, t = 10 mm, P_{max}= 100000 N, R = 0.1, w = (50, 65, 80) mm. Under cyclic axial loading with constant amplitude the failure behaviour is theoretically considered by assuming the same material parameters as those mentioned above.

In the residual life assessment, the stress intensity factor is calculated through Eqs. (3)-(11), whereas the number of loading cycles is estimated by applying Eqs. (2a)-(2b) for both depth and surface crack directions. The effects of thickness and width are shown (as numbers of loading cycles to failure against crack length) in Figs. 4a and b, respectively. It should be noted that calculated curves 1a, 2a, 3a and corresponding 1b, 2b and 3b (see Fig. 3) represent the fatigue estimations for the depth and surface crack length direction, respectively.

The comparisons presented in Fig. 3 show that thickness and width have significant impact on the fatigue strength of the plate with quarter-elliptical corner crack. Thus, the developed computational model enables a quick identification of the most influent parameters, in order to reach an optimal design solution, as well as to timely predict fatigue failure caused by initial quarter-elliptical corner crack during mandatory inspections and controls.



(a)

(b)

Figure 3. Crack length against number of loadng cycles to failure: (a) the effect of thickness of the plate (w = 65 mm, 1 – t = 12 mm, 2 – t = 15 mm, 3 – t = 18 mm), (b) the effect of width (t =10 mm, 1 – w = 50 mm, 2 – w = 65 mm, 3 – w = 80 mm).

CONCLUSIONS

In the present paper, the fatigue failure behaviour of a quarter-elliptical corner crack emanatig from a hole is theoretically examined. Such a fracture mechanics based investigation takes into account the stress analysis and the residual life estimation. Under cyclic loading, in order to describe the stress-intensity behaviour analytical approach is employed. Further, the fatigue life to failure is assessed by means of the stress-dependance crack growth law. A good correlation between the crack growth calculations and available experimental data verifies that proposed computational model can be employed for reliable strength analysis of the quarter-elliptical corner crack located at a hole.

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References

[1] Elber, M. (1971). The significance of fatigue crack closure. In: Damage Tolerance in Aircraft Structure. ASTM STR 486, p. 230-242.

[2] Walker, K. (1970). The effect of stress ratio during crack propagation and fatigue for 2024-T3 and 7075-T6 aluminum. In: Effect of environment and complex load history on fatigue life. ASTM STR 462, p. 1-14.

[3] Kujawski, D. (2001). A new $(\Delta K^{+} K_{max})^{0.5}$ driving force parameter for crack growth in aluminum alloy. International Journal of Fatigue, vol.23, p.733-740.

[4] Glinka, G., Robin, C., Pluvinage, G., Chehimi, C. (1984). A cumulative model for fatigue crack growth and the crack closure effect. International Journal of Fatigue, vol.6, no. 1, p. 37-47.

[5] Atluri, S.N., Nishioka, T. (1986). Computational methods for three-dimensional problems of fracture. In: Atluri, S.N. (editor), Computational methods in Mechanics of Fracture, Chapter 7, North Holland, New York p. 230-287.

[6] Raju, I.S., Newman, Jr. J.C. (1979). Stress intensity factors for a wide range of semi-elliptical surface cracks in finite-thickness plates. Engineering Fracture Mechanics, vol. 11, p. 817-829.

[7] Tracey, D.M. (1974). Finite element for three-dimensional elastic crack analysis. Nuclear Engineering and Design, vol. 26, p. 282-290.

[8] Boljanović, S., Maksimović, S., Djurić, M. (2016). Fatigue strength assessment of initial semi-elliptical cracks located at a hole. International Journal of Fatigue, vol.92, p. 548-556.

[9] Newman, Jr. J.C. (1976). Predicting failure of specimens with either surface cracks or corner cracks at holes. NASA TN-D-8244, NASA Langley Research Center, Hampton, Va., USA.

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