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NUMERICAL METHODS FOR DETERMINATION THE ELASTIC STRESS AND DEFORMATIONS IN ROLLINGS BEARINGS

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Abstract: The modern methods of mathematical theory of mathematical theory of elasticity permit to solve a large series of the problematic of bearings. In this study is presented the results of the use of plane theory of elasticity for study of the state of tensions in intern inner.The projection of the bearings elements, special the rolling bearings and the roller way is very important. It was studied the aspect of stresses in rolling with half-space method, finite elements (contact element), MathCad programmes. The compression of a cylinder in contact "nonconformist" with two surfaces, who are in opposition at the extremity of roles, can be analyses.
Keywords: numerical methods, finite elements, half-space, mathcad, bearings

INTRODUCTION

One of the best methods to determinate the stresses are the numerical methods. In this application we use same different numerical methods for determination the state of bearings stresses, very important for your projections.

The modern methods of mathematical theory of mathematical theory of elasticity permit to solve a large series of the problematic of bearings. In this study is presented the results of the use of plane theory of elasticity for study of the state of tensions in intern inner. The system is compound by the intern inner and the motor shaft acting by concentrated force applied on rolling way.

Classical elastic contact stress theory concerns bodies whose temperature is uniform. Variation in temperature within the bodies may, of itself, give rise to thermal stresses but may also change the contact conditions through thermal distortion of their surface profile.

The skill of machines tools is based in very large measure on the reliability of the bearings.

HALF-SPACE METHOD - Uniform pressure applied to a polygonal region

We shall consider in this section a uniform pressure p applied to a region of the surface consisting of a straight-sided polygon, as shown in fig (1.a). It is required to find the depression at a general point B (x, y) on the surface and the stress components at a subsurface point A(x, y) ,BH_1, BH_2, etc, are perpendiculars of lengths h_1,h_2, etc. onto the side of polygon DE,EF respectively. The loaded polygonal is then made up of the algebraic addition of eight right angle triangles:

$$EFG = [BEH_1+BEH_2+BFH_2+BFH_3] - [BDH_1+BDH_4+BGH_3+BGH_4] \quad (1)$$

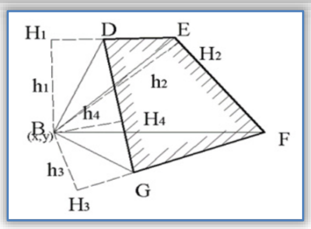
A similar breakdown into rectangular triangles would have been possible if B had lain, inside the polygon a typical triangular area is shown in fig (1.b)

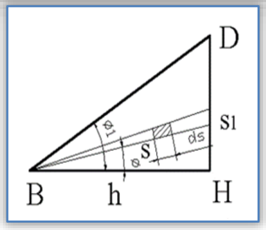
$$(\bar{u}_y)_B = \frac{1-\nu^2}{\pi E} p \int_0^{\phi_1} d\phi \int_0^{s_1} ds = \frac{1-\nu^2}{\pi E} p \int_0^{\phi_1} h \sec\phi \, d\phi = \frac{1-\nu^2}{\pi E} p \frac{h}{2} \ln\left(\frac{1+\sin\phi_1}{1-\sin\phi_1}\right) \quad (2)$$

The total displacement at B due to a uniform pressure on the polygonal region DEFG can then be found by combining the results of equations (2) for the eight constitutive triangles. The stress components at an interior point A(x, y, z) below B can be found by integration of the stress components due to a point force given by known equation but the procedure is tedious [2]

The effect of a uniform pressure acting on rectangular area 2a*2b has been analysis in detail by Lowe (1929). The deflection of a general point (x, y) on the surface is given by:

$$D = \frac{\pi E}{1-\nu^2}\frac{\bar{u}_z}{p} = (x+a)\ln\left[\frac{(y+b)+\left((x+a)+(x+a)^2\right)^{1/2}}{(y-b)+\left((y-b)+(x+a)^2\right)^{1/2}}\right] + (y+b)\ln\left[\frac{(x+a)+\left((y+b)^2+(x+a)^2\right)^{1/2}}{(x-a)+\left((y+b)^2+(x-a)^2\right)^{1/2}}\right] + (x-a)\ln\left[\frac{(y-b)+\left((y-b)^2+(x-a)^2\right)^{1/2}}{(y+b)+\left((y+b)^2+(x-a)^2\right)^{1/2}}\right] + (y-b)\ln\left[\frac{(x-a)+\left((y-b)^2+(x-a)^2\right)^{1/2}}{(x+a)+\left((y-b)^2+(x+a)^2\right)^{1/2}}\right] \quad (3)$$



a)


b)

Figure 1: a) Uniform pressure;
b) A typical triangular area

Expressions have been found by Lowe (1929) from which the stress components at a general point in the solid can be found. Lowe comments on the fact that the component of shear stress □xy has a theoretically infinite value at the corner of the rectangle. Elsewhere all stress components are finite. On the surface at the centre of the rectangle:

$$[\sigma_x]_0 = -p \{2\nu + (2/\pi)(1-2\nu)\tan^{-1}(b/a)\}$$
$$[\sigma_y]_0 = -p \{2\nu + (2/\pi)(1-2\nu)\tan^{-1}(a/b)\} \quad (4)$$
$$[\sigma_z]_0 = -p$$

These results are useful when a uniform loaded rectangle is used as a 'boundary elements' in the numerical solution of more general contact problems.

The elastic deformation in a point (x, y) make by the uniform distribute pressure from the rectangular surface (2a*2b) will be Figure 2.

$$\delta' = \frac{p}{\pi E}\int_{-a}^{a}\int_{-b}^{b}\frac{dxdy}{\left[(y-y_1)^2+(x-x_1)^2\right]^{1/2}} \quad (5)$$

By integrations the equation effect:

$$\delta = \frac{pD}{\pi E'} \quad (6)$$

where: - the displacement D, is calculated by the formula (2)

The expression δ represent the elastic deformation in the point (x, y) make by the uniform pressure p, distribute from the rectangular surface (2a*2b). If the contact surface is divided in a number of rectangular equal surface, the total deformation in point (x, y) make by contribution of the diverse uniform rectangular surface load, in the contact surface made by numerical evaluated.

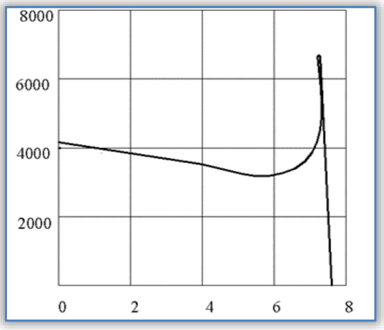


Figure 2: Uniform distribute pressure from the rectangular surface

The total deformation make by the uniform load from the rectangular surface in the inside of the con

$$\delta_i = \frac{1}{\pi E}\sum_{j=1}^{n} p_j D_{i,j} \quad (7)$$

The results obtained by this method using for the contact of cylindrical bearings N2256 is giving in application [2].

FINITE ELEMENT METHOD

The compression of a cylinder in contact "nonconformist" with two surfaces, who are in opposition at the extremity of roles, can be analyses satisfactory (Figure 3).

A compression force on the unity of length we give a hertz distribution of pressure in O_1 equal with:

$$p = \frac{2P}{\pi a_1}\left(1-\frac{x^2}{a^2}\right)^{\frac{1}{2}} \quad (8)$$

$$a_1^2 = 4PR/\pi E_1^* \quad (9)$$

E_1^* - Young modulus

The tensions in A are given by the contribution:

- the tensions given by the hertz distribution in O_1
- the tension given by the pressure in O_2, may be considered as for a concentrate force P
- the biaxial tension given by the equation

$$\sigma_1 - \sigma_2 = P/\pi R \quad (10)$$

Assembly the three contributions, we obtain:

$$\sigma_x = \frac{P}{\pi}\left[\frac{1}{R} - \frac{2(a_1^2+2z^2)}{a_1^{2(}(a_1^2+z^2)^{\frac{1}{2}}} + \frac{4z}{a_1^2}\right]$$
$$\sigma_z = \frac{P}{\pi}\left[\frac{1}{R} - \frac{2}{2R-z} - \frac{2}{(a_1^2+z^2)^{\frac{1}{2}}}\right] \quad (11)$$

The real cylinders are finite length and the important deviations at the Hertz theory appear to their end.

- The description of the construction solution

With the finite element program ANSYS use plane elements (triangular, rectangular and contact elements 48 we realized in the case of a cylindrical roles one other profile, a Lundberg modified profile.

- The advantage of the proposed solution

The programmer utilized the contact elements and has on view the relative positions of the two surfaces.

The finite elements are triangular, rectangular and contact elements, where the base is make by the nodes of the twice surfaces target and by the last contact with the first surface-contact.

These elements of contact are finite elements that utilize one pseudo-element as the techniques of establish of the two surface of contact.

Also they equalize the forces who existing in the contact nodes between two surfaces (in reality this perfect contact is not real).

The compatibility of the contact is one combination at a penalization functions and a Lagrange multipliers used in program.

In the fig (4) is presented the discretisation and the deformation of the case of the contact of the roles with right generator and in fig (3) is presented the discretisation and deformation in cylindrical right roles

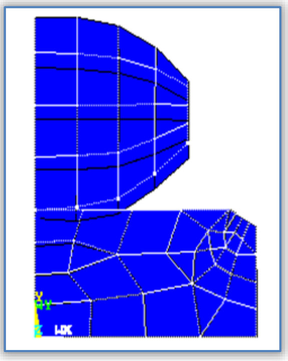


Figure 3: Discretisation and deformation in cylindrical right roles

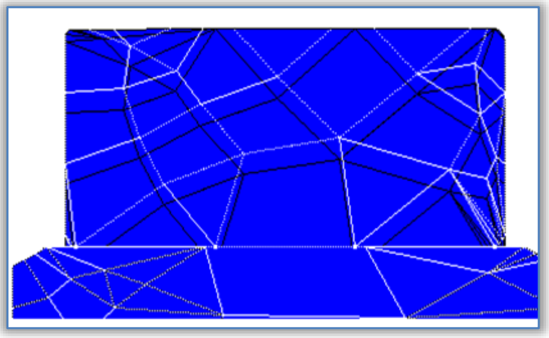


Figure 4: Discretisation and deformation in cylindrical roles with Lundberg modified profile

In (Figure 5) and (Figure 6) we can observe the distribution of stresses in two type of roles – cylindrical right roles (fig 5) and cylindrical roles with Lundberg modified profile propose by author (Figure 6).We can also observe that the stresses at the end of the roles are large small in the second case [4].

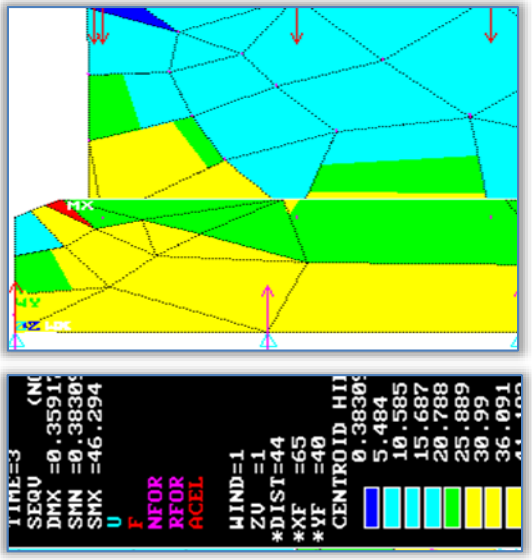


Figure 5: Distribution of tensions in cylindrical right role

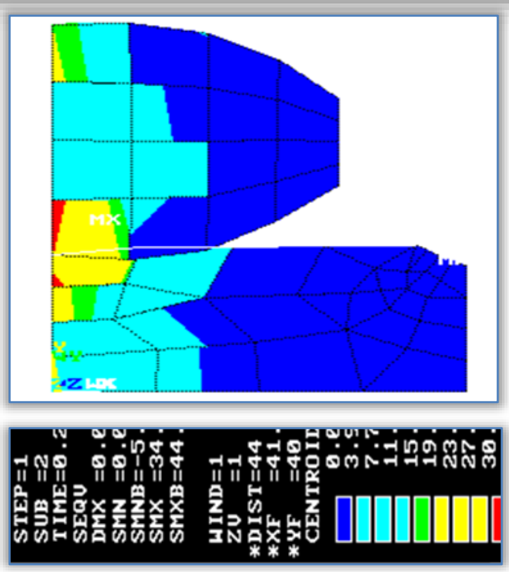


Figure 6: Distribution of stresses in cylindrical roles with Lundberg modified profile

HERTZIAN CONSTANTS COMPUTER ASSISTED PROCESS DESIGN USIGN MATHCAD

For the understanding the hertzian models, it was study first the constituent equations for the vertical displacement u_z.

The hypothesis I is associate to establishment the path in a median elastic plane dependent by the curves of the conjugated surface and the elastics contacts of the two surface cylinders the account of contact verifying the consigns equations (Figure 7).

$$(z_0+u_0)+(z_1+u_1)=h,\ h=h_0+h_1 \qquad (12)$$

The external point of the contact, verifiable the non-equation

$$(z_0+u_0)+(z_1+u_1)<h \qquad (13)$$

Take by P_z (x, y), the distribution of the contact pressure we have:

$$u_i' \frac{1}{\pi E_i^*}\iint \frac{P_z(\xi,\eta)}{r}\,d\xi\, d\eta,$$

$$E^*{}_i=\frac{E}{1-\nu^2} \quad I=0\ldots n \qquad (14)$$

where: E_i, ν_I - is the Young and Poisson coefficients of this two materials

The description of the construction solutionE_i, ν_I - is the Young and Poisson coefficients of this two materials.

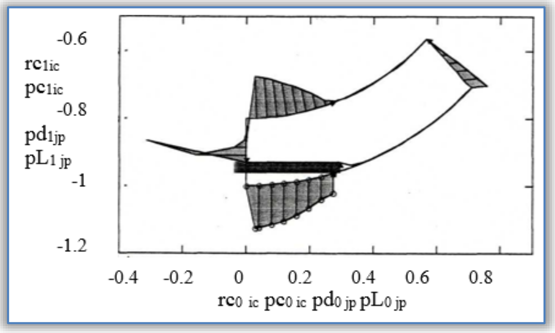


Figure 7: Pressure of contact distribution

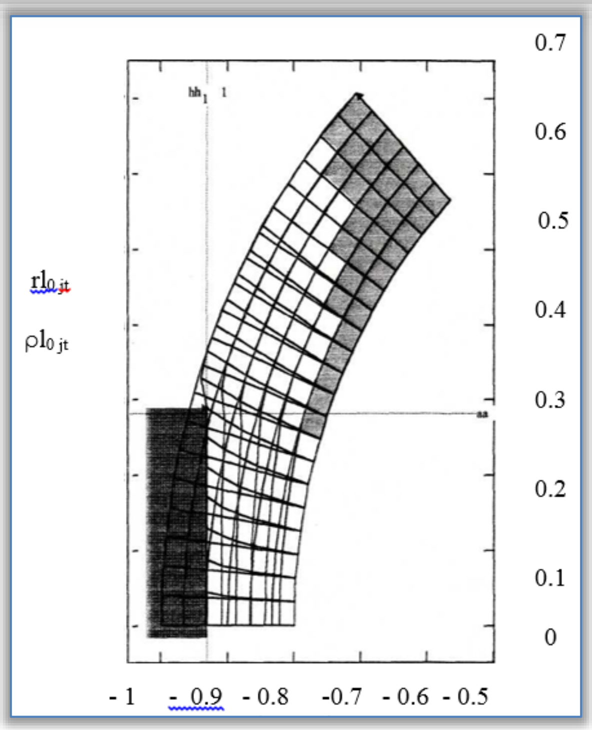


Figure 8: Flatten in the profile plane

For construct an imagine of the sliding in the hertz plain I am stimulated one of two sphere by the plane structural complex by beam elements, for 7 radial level and twenty one angular (266 elements, 21*7=147 nodes).

To fix to structure embed for the contour 0… 20, 41, 62, 146 radial sliding for the 21, 42…126 nodes.

It rested that the slides for the contact plane and at the same time is making be determinates the pressure of contact distribution (Figure 7).

The impose reshuffle of force is corresponding of flatten in the profile plane (Figure 8) [3].

CONCLUSIONS

The numerical methods are one of the best methods to determinations the stress in the roles and rolling ways. It is very important the projects of the profile of roles for determinations of the state of stress.

The numerical methods are one of the best methods to determinations the tensions in the rolls and rolling ways. It is very important to now, because the project of the profile of roll is very outstanding for determinations of the state of tensions. It results that the static model Lundberg modified had to be performed carefully, from the upper mentioned details.

Our work proposed also a model of analysis of the tensions and contact situation at the roller bearings And an analysis of the contact situation, created on the ANSYS program structure.

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