ACTA TECHNICA CORVINIENSIS
– Bulletin of Engineering
Tome IX [2016], Fascicule 2 [April – June]
ISSN: 2067 – 3809

[1.]Aleksandar KOŠARAC, [2.] Milan ZELJKOVIĆ, [3.] Cvijetin MLAĐENOVIĆ,
[4.]Aleksandar ŽIVKOVIĆ, [5.] Saša PRODANOVIĆ

STATE SPACE MODELING FROM FEM MODEL USING BALANCED REDUCTION

[1,5.] University of East Sarajevo, Faculty of Mechanical Engineering, East Sarajevo, BOSNIA & HERZEGOVINA
[2-4.] University of Novi Sad, Technical Faculty of Technical Sciences, Novi Sad, SERBIA

Abstract: This paper shows discretized model of main spindle working unit module obtained by ANSYS software. Based on the conducted modal analysis for first ten bending modes within frequency range from 0 to 10 kHz, main shapes and natural frequency (eigenfrequency) have been determined, and modal matrix was formed. Obtained results were afterwards used to determine model dynamic behavior in the state space using MATLAB. State space model was further used as reference one for modal reduction whereby balanced reduction method was applied. Frequency response of full model (all oscillatory modes) and reduced model for direct FRF and cross FRF are presented through Bode diagram. Reliability of the model is verified by comparing the impulse response of full model and reduced model in time domain.
Keywords: natural frequencies, model order reduction, modal reduction, balanced reduction, state space

INTRODUCTION

Considering that finite element models may have high degree of freedom, MKE analysis may require more time, especially causing problems when analysis are often repeated during the designing process. Therefore, methods simplifying finite elements models are developed in the manner that the most important characteristics of the original dynamic system are included.

Model reduction is a method used to decrease the time required for simulation when finite element model are used and to obtain simplified model preserving at the same time wanted dynamic characteristics of original system. The task of the model reduction is to replace mathematical model of the system or of the process with the one (model) that is far smaller from the original one but still provides input/output relationships of the system or of the process. This paper is the sequel of the research presented in [4] which showed modal analysis by FEM on the working unit module main spindle, where based on the first ten bending vibration modes modal matrix was determined through analyzing displacement in ten measuring points. Afterwards, modal reduction was conducted whereby two methods of ranking of individual mode contribution to the overall frequency response were used, as follows: dc gain and peak gain. Which one, out of two, will be used, depends entirely whether damping ratio ζ has unique value or different value for individual modes. This paper presents SISO model (Single Input Single Output) wherein, for mode ranking, dc gain and peak gain might be used, but also other methods, such as balanced reduction. Hereby concepts of controllability and observability will be used for modes ranking. The method of reducing models "balanced reduction" is applied, using both ranking concepts simultaneously. In this paper FEM modal analysis results and balanced reduction technique were used to obtain a low order state space model of main spindle. Response of the main spindle due to excitation will be obtained using reduced number of vibration modes.

Equivalent dynamic model of main spindle with measurement points analyzed in this paper, is shown in figure 1, while discretized model of main spindle is shown in figure 2.

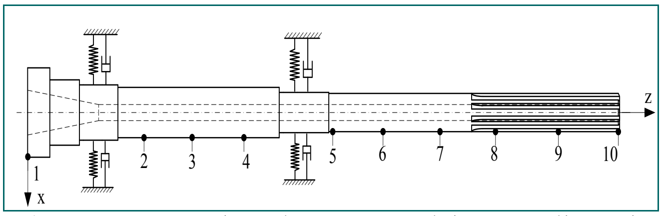

Figure. 1. Equivalent dynamic model of spindle with measurement points [4]

© copyright Faculty of Engineering – Hunedoara, University POLITEHNICA Timisoara

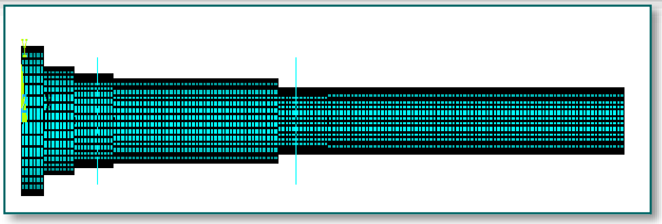

Figure 2. Discretized model of main spindle [4]

Main spindle is supported by the two sets of angular contact ceramic ball bearings in front, SKF S7011 CD/HCP4A and two sets of angular contact ceramic ball bearings in rear SKF 7008 CD/HCP4A, installed back to back [5], [6]. SOLID186 a higher order 3-D 20-node solid element is used to simulate main spindle, and spring damper element COMBIN 14 is applied to simulate the elastic support of the two set of bearings. Eighteen elements were set along the circumferential direction of the spindle on each set of bearing, simulating rolling elements. Since the inertial force and the thermal expansion of bearing elements affects the balls, an uneven distribution of contact forces appears and an uneven contact angle change occurs. The consequence of the aforementioned is the uneven distribution in bearing stiffness, depending on the position of the ball [6]. Values of bearing stiffness depending on the preload and main spindle dimension in are provided in [4], [5]. The first ten eigenvalues for bending motion of main spindle extracted using Block-Lanczos method are shown in Table 1.

Table 1. Eigenvalues for bending motion of main spindle, Hz

f_1	f_2	f_3	f_4	f_5
154,37	926,92	1641	2351,8	2538,1
f_6	f_7	f_8	f_9	f_{10}
4533,1	5687,4	6972,7	8228,9	9637,4

CONTROLABILITY AND OBSERVABILITY CRITERIA

There are different definitions of controllability and observability for state space system, described by equation (1).

$$\dot{x} = Ax + Bu \qquad y = Cx \tag{1}$$

According to [3] system is controllable if there is an input „u“ that can move the system from some arbitrary state x_1 to another arbitrary state x_2 in a finite time. Similar, the system is observable if the initial state x_0 of a system can be inferred from knowledge of the input u and the output y over a finite time (0,t).

Controllability as a measure of interaction between the input and the states involves the system matrix A and the input matrix B. Observability, as a measure of interaction between the states and the output involves the system matrix A and the output matrix C.

There are several criteria that determine whether a system is controllable and observable. A linear time-invariant system (A,B,C), with s inputs is completely controllable if and only if the NxsN matrix

$$\left[B \; AB \; A^2B \; \cdots \; A^{n-1}B \right] \tag{2}$$

has rank N. A linear time-invariant system (A,B,C) with r outputs is completely observable if and only if the rNxN matrix of has rank N.

$$\begin{bmatrix} C \\ CA \\ CA^2 \\ \cdots \\ CA^{n-1} \end{bmatrix} \tag{3}$$

There are two disadvantages of this criterion. The first one is that the criteria is suitable for operating with a small dimensions system only. In fact, it is to answer the question whether a system of controllable or observable A^{n-1} should be found, which is obviously a problem from the standpoint of numerical data processing with a larger system.

Another disadvantage is that the answer to the question of whether the system is controllable or observable, is only a "yes" or "no", which may not be the case if the application of other criteria.

Another criterion in determining the controllability and observability uses gramians to determine system properties.

Gramians controllability and observability can be determined from differential equations

$$\dot{W}_c = AW_c + W_cA^T + BB^T \qquad \dot{W}_o = A^TW_o + W_oA + C^TC \tag{4}$$

whose solutions is a time-dependent matrix. For a stable system, stationary solutions are obtained assuming $\dot{W}_c = \dot{W}_o = 0$ whereby differential equations become equations, known as Lyapunov's equations

$$AW_c + W_cA^T + BB^T = 0 \qquad A^TW_o + W_oA + C^TC = 0 \tag{5}$$

where W_c is controllability Gramian, and W_o observability Gramian.

Another definition of controllability and observability involves gramians W_c and W_o, the solutions to the Lyapunov equation (5) defined as

$$W_c = \int_0^\infty e^{A\tau}BB^Te^{A^T\tau}d\tau \qquad W_o = \int_0^\infty e^{A^T\tau}C^TCe^{A\tau}d\tau \tag{6}$$

If the solutions $W_c(t)$ and $W_o(t)$ are non – singular, then system is controllable, i.e. observable.

In modal coordinates the diagonal entries of the controllability and observability gramians are as follows [2], [3]:

$$w_{ci} = \frac{\|B_i\|_2^2}{4\varsigma_i\omega_i} = \frac{\left\|\begin{bmatrix} 0 \\ F_k z_{nki} \end{bmatrix}\right\|_2^2}{4\varsigma_i\omega_i} = \frac{F_k^2 z_{nki}^2}{4\varsigma_i\omega_i} \qquad (7)$$

$$w_{oi} = \frac{\|C_i\|_2^2}{4\varsigma_i\omega_i} = \frac{\left\|\begin{bmatrix} z_{nji} & 0 \end{bmatrix}\right\|_2^2}{4\varsigma_i\omega_i} = \frac{z_{nji}^2}{4\varsigma_i\omega_i}$$

while Hankel singular values are obtained from

$$\gamma_i \cong \frac{\|B_i\|_2 \|C_i\|_2}{4\varsigma_i\omega_i} \qquad (8)$$

For systems that have relatively small values of the damping ratio ζ, gram matrix is diagonal dominant, which means that the element outside the main diagonal have significantly lower values than elements which belong to the main diagonal.

The syntax for the MATLAB function *balreal* which produces a balanced realization of the linear time-invariant model with equal and diagonal controllability and observability gramians is:

$$[sysb, g, T, Ti] = balreal(sys) \qquad (9)$$

where *sysb* is new balanced system, and *g* is diagonal of the joint gramian. Diagonal entries of the joint gramian g are squares of the Hankel singular values of the system. After the balanced gramian diagonal terms are sorted in descending order, *modred* function, option *mdc* or *del* can be applied to eliminate states with the lowest joint controllability/observability.

RESULTS AND DISCUSSION

Figure 3 and 4 shows direct FRF (X1/F1) and cross FRF (X10/F1), where all ten modes were included. Modes are sorted based on the dc gain whereby damping ratio has a uniform value ζ = 0.001. Individual modes contribution was shown on both figures also.

FRF shown in figure 3 does not contain first two natural frequencies (154,37 Hz and 926,92 Hz) i.e. modal contribution of the first two modes is smaller than contribution of modes three, four, five and nine. Explanation how to calculate modal contribution and preform ranking of all modes for this case is provided in [4]. Reasons for this can be found in figure 5 and figure 6, where it can be seen that the main spindle tip displacement on the first two natural frequencies is negligible. On the other hand, figure 4 represents cross FRF, i.e. displacement of measurement point 10 (figure 1), where first two natural frequencies are present. From figures 5 and 6 it can be noticed that point 10 has maximum displacement. Also, figures 7 and 8 show that at third and fourth natural frequencies (1641 Hz and 2351,8 Hz) both measurement points (1 and 10) have significant displacement. As a result both frequencies are present in direct and cross FRF, as can be seen from figures 3 and 4.

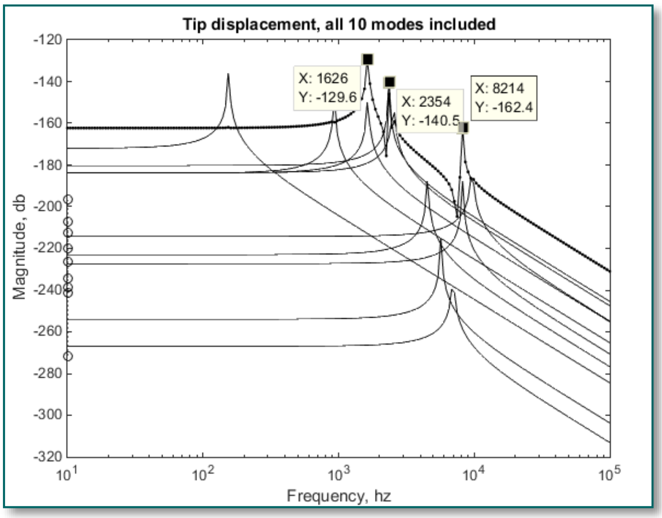

Figure 3. Direct FRF, all 10 modes included

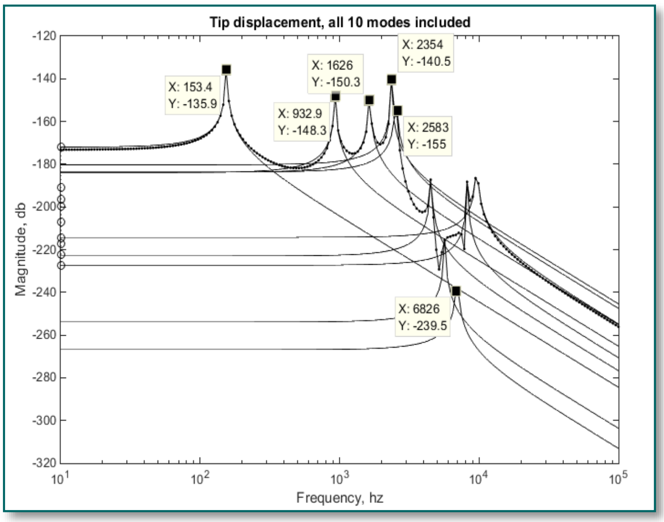

Figure 4. Cross FRF (X10/F1)– all ten modes include

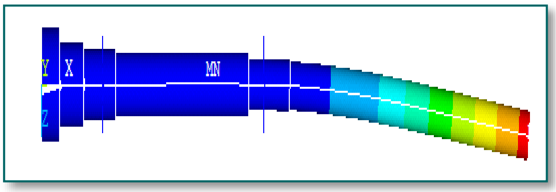

Figure 5. First bending mode 154,37 Hz

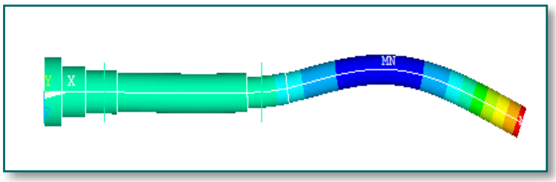

Figure 6. Second bending mode 926,92 Hz

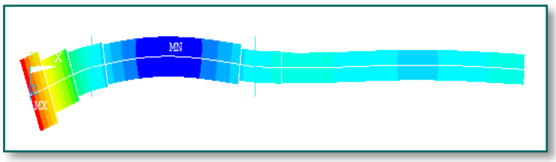

Figure 7. Third bending mode 1641 Hz

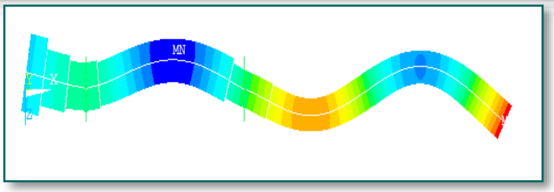

Figure 8. Fourth bending mode 2351,83 Hz

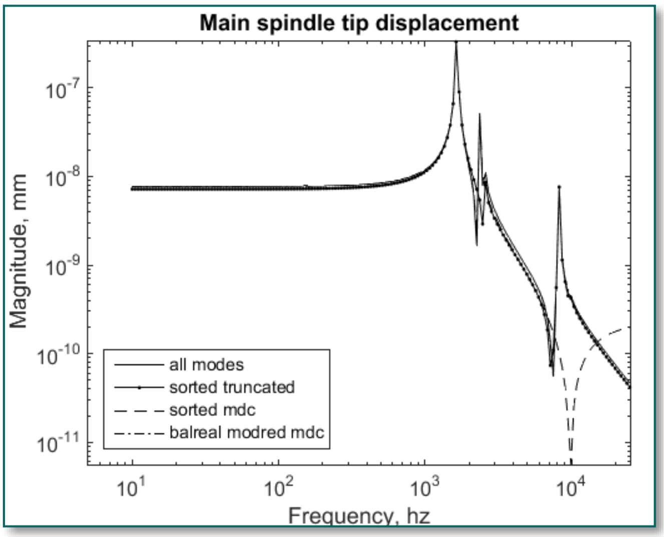

Figure 9. FRF for full model and four modes included, *balreal modred mdc* option used

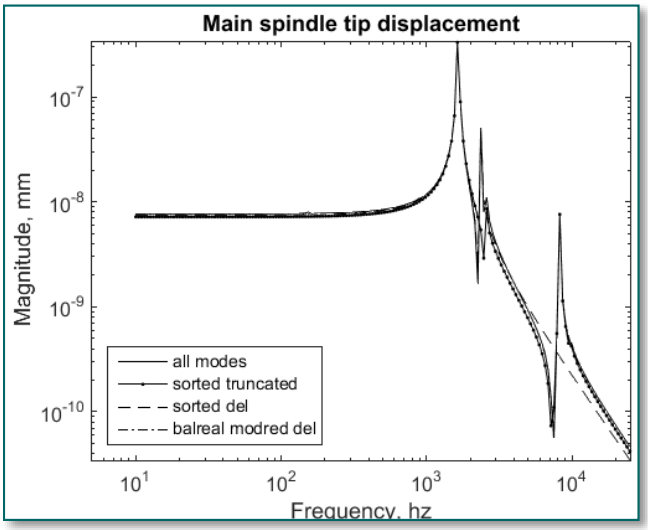

Figure 10. FRF for full model and four modes included, *balreal modred del* option used

Figures 9 and 10 show direct FRF where full model and first four modes were included, with different reductions technique used. Frequency response plot in figure 9 shows:

- Full model (all oscillatory modes)
- Sorted truncated, where dc gain was used for mode ranking, after that least significant modes were deleted
- Sorted mdc where dc gain was used for mode ranking and MATLAB function modred with mdc option to reduce
- Balreal modred mdc, where MATLAB function modred was used with mdc option to eliminate states with the lowest joint controllability /observability,

while frequency response plots in figure 10 shows:

- Full model (all oscillatory modes)
- Sorted truncated, where dc gain was used for mode ranking, after that least significant modes were deleted
- Sorted del, where dc gain was used for mode ranking and MATLAB function modred with del option to reduce
- Balreal modred del, where MATLAB function modred was used with del option to eliminate states with the lowest joint controllability/observability.

It can be noticed that in the high frequency portion, magnitude is rising when *modred* function *mdc* option is used which is not a case with *del* option. But, when modes are sorted using balanced system controllability and observability gramians there is no rising magnitude regardless which option was used. Comparison of the impulse responses in time domain is shown in figure 11.

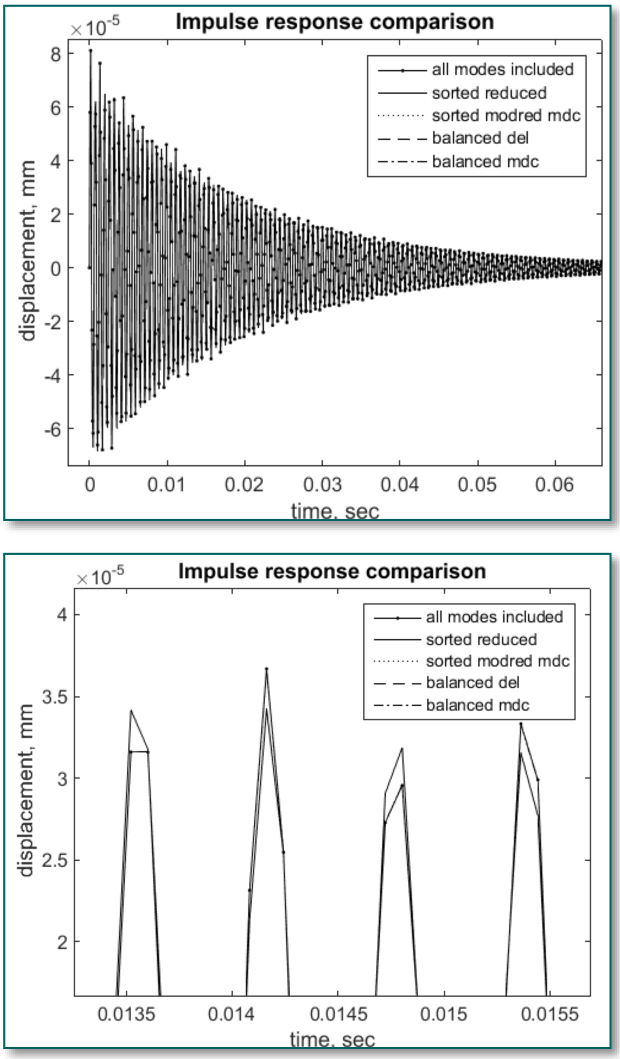

Figure 11. Impulse response comparison, full model and four modes included

Impulse response contains full model and four modes included with different reduction options used. It can be seen that there are no significant differences in the time domain response.

CONCLUSIONS

This paper shows, on the main spindle example, how the model can be transformed from finite elements into state space representation i.e. how to take the results of FEM and reduce the model size extracting lower order state space model in MATLAB (model reduction). Therefore, the goal is not only to obtain reliable dynamic model, since such a model is FEM model.

In the sense of the written, working on the subject above resulted in the following:

» Convert a large finite element model (“large model” assumes model with thousands of hundreds of DOF) to a smaller MATLAB model which still provides correct response for the forcing input, i.e. still maintaining the input / output relationship.

» Modal reduction was made by using balanced reduction technique.

» A reduced solution provides very reliable dynamic of the model with a significant reduction in number of states. Reduced mode can be further inserted into a more complex control system model and used to find system dynamics.

Note

This paper is based on the paper presented at The Vth International Conference Industrial Engineering and Environmental Protection 2015 – IIZS 2015, University of Novi Sad, Technical Faculty „Mihajlo Pupin”, Zrenjanin, SERBIA, October 15-16th, 2015, referred here as[7].

REFERENCES

[1] Garitaonandia, I, Sabalza, X, Fernandes, M.H, Munoa, J, Hernandez, M.J., Albizuri, J.: State space modelling of a centerless grinding machine, EUROCON 2005 - The International Conference on "Computer as a tool", Serbia and Montenegro, Belgrade, November 20-24,2005.

[2] Gawronski, W,K.: Dynamics and Control of Structures, A modal approach, Springer, 1988

[3] Hatch, M. R.: MATLAB and ANSYS, Chapman & Hall/CRC, 2001.

[4] Košarac, A., Zeljković M., Mlađenović, C., Živković, A.: Create SISO state space model of main spindle from ANSYS model, MMA 2015 Flexible Technology, 12th International Scientific Conference Novi Sad, Serbia, September 25-26, 2015, 37-41

[5] Zeljković, M.: Sistem za automatizovano projektovanje i predikciju ponašanja sklopa glavnog vretena mašina alatki, Doktorska disertacija, Fakultet tehničkih nauka, Novi Sad, 1996.

[6] Živković, A., Zeljković, M., Tabaković, S., Milojević, Z.: Mathematical modelling and experimental testing of high – speed spindle behaviour, The International Journal of Advanced Manufacturing Technology, (2015) 77: 1071-1086.

[7] Aleksandar Košarac, Milan Zeljković, Cvijetin Mlađenović, Aleksandar Živković, Saša Prodanović, State space modeling from fem model using balanced reduction, The Vth International Conference Industrial Engineering and Environmental Protection 2015 – IIZS 2015, University of Novi Sad, Technical Faculty „Mihajlo Pupin”, Zrenjanin, SERBIA, October 15-16th, 2015

ISSN:2067-3809

copyright ©
University POLITEHNICA Timisoara,
Faculty of Engineering Hunedoara,
5, Revolutiei, 331128, Hunedoara, ROMANIA
http://acta.fih.upt.ro