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STOCHASTIC MODELING OF HONING PROCESSES

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Abstract: More accurate description of abrasive manufacturing procedures can be done stochastic methods that is why their application are advantageous. The author have elaborated the stochastic mathematical model of abrasive microcutting systems and processes of the tools with undetermined edge-geometry and many edges that makes description of e.g. honing, grinding possible. The system of mathematically formulated relationships corresponds to the experimental observations. The elaborated method is applicable also to describe the abrasive wear processes at grinding or at machining. This method provides the ability to calculate and design the statistical parameters of the machined surface and the process.
Keywords: abrasive manufacturing procedures, stochastic methods, honing

INTRODUCTION

The figure 1 demonstrates schematically the interaction of an optional abrasive tool and a rough workpiece. Validity conditions of the model are in publication [1, 2].

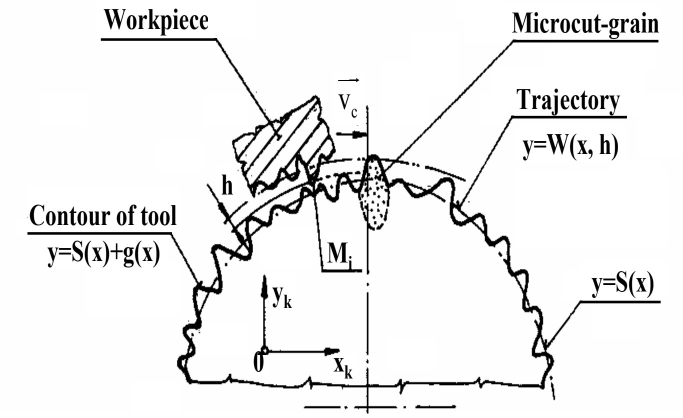

Figure 1. Stochastic model. Interaction of a multi-edge tool and the workpiece ($\vec{v}_c$ is the cutting speed).

The most important conditions are as follows.

- The microgeometrical traces formed on the machined surface depending on the shape size, number, distribution of the grains which create them and the technological data (average depth of cut, trajectory, etc.).
- There are not built up edges on the tool. In most cases the material of the workpiece is cast iron or hardened steel.
- Wear of the tool is ratherly slow in the case of super hard cutting tools, the process can be pretend to stationary.
- Examinations are done in the standing coordinate system fixed to the tool.

INTERACTIONS OF TOOL AND WORKPIECE. STOCHASTIC MODEL

The trajectory of the point M_i of the workpiece depends on kinematics of the machine, on the applied technological parameters (components of cutting speed), thus on the actual realisation of the forming mechanism. The point M_i is situated in h height above a designated "zero-level". This height will be worked off by the designated cutting grain. Zero-level: h=0. Other precondition: $y_0=0$. The task is to determine the probability of connections of the most protruding cusps on the workpiece with the most protruding grains, thus the expected height of developing roughness. The equation of trajectory of the point M_i in the orthogonal system of axes taken according to point d is:

$$y=W(x;h), \qquad (1)$$

where the sindow variables are x,h.

The profile of the tool is:

$$y=S(x)+g(x), \qquad (2)$$

where S(x) is a deterministic function describing the macro-form of the tool, g(x) is a stochastic stationary formula that characterises the peakness of the grains. The trajectory that belongs to the point M_i is passing through without any connection over the tool-profile. In this case:

$$W(x;h)>S(x)+g(x). \qquad (3)$$

The probability of completion of this relation can be expressed by such a stochastic function that equals to the probability of the skip-free state of the

$$\Psi(x;h)=S(x)+g(x)-W(x;h) \qquad (3/a)$$

stochastic function at the given "zero-level". Selection of such a function is a complicated task, however it can be simplified since the tool surface (the height of the protrusion peaks) can correctly be characterised by the Gaussian distribution for honing tools with super-hard grains, according to practical experiences. The probability of the skip-free state is the following [2, 3]:

$$P(h)=exp\left[-\int_{x_{min}}^{x_{max}}\int v f_1(y;v)dvdx\right], \qquad (4)$$

where $f_1(y;v)$ is the density function that characterises the peakness of the tool. The relation between the ordinate values y and variable v is:

$$v=\frac{dy}{dx}$$

The latter differential quotient expresses the form, running off (incline or direction factor) of the grain-edge. The interval of the skip's examinations: x_{max}-x_{min}, where the tool-workpiece connections are possible at all; $f_1(y;v)$ may be expressed by the stochastic characteristics of the g(x) micro-profile of the abrasive tool given by the f(y;v) density function.

It is conceivable that if $y=\Psi(x;\ h)$, then $g(x)=y-S(x)+W(x;h)$. Similarly:

$$\frac{\partial \Psi(x;h)}{\partial x}=v, \text{ then } \frac{\partial g(x)}{\partial x}=v-\dot{S}(x)+\dot{W}(x;h),$$

where

$$\dot{S}(x)=\frac{\partial / S(x) /}{\partial x} \text{ and } \dot{W}(x;h)=\frac{\partial W /(x;y)/}{\partial x}.$$

Differentiation at the x=0 spot will lead to the following relation:

$$f_1(0;v)=f[W(x;h)-S(x);\ v-\dot{S}(x)+\dot{W}(x;h)]. \quad (5)$$

During machining, the cutting tool turns into contact with the workpiece several times. During the previous operation a characteristic micro-topography of the workpiece has already been formed, which depends on the applied machine-tool kinematics, the tool and the set of technological characteristics,s thus on the so-called "forming mechanism". The surface after the rough machining is characterised by the $P_0(h)$ distribution function.

The number of repeated connections of the tool and the workpiece during the machining is n. At the i-th touch the $f_{1i}(0;v)$ density function holds good. Conversely, the depth of cut will be changed due to displacements and elastic deformations. If the point M_i, will contact the tool k times, then:

$$P(h)=P_0(h)exp\left[-k\sum_{i=1}^{n}\int_{x_{min}}^{x_{max}}\int_{0}^{\infty} vf_{1i}(0;v)dvdx\right]. \quad (6)$$

The "zero-level" of P(h) and that of $P_0(h)$ is equivalent, of course.

Consequently, the height-distribution of the workpiece's micro-roughness should be expressed in the plane that is common with the tool-profile, applying the stochastic function that describes the tool-profile. Substituting the stochastic function that designates the tool into the equation (6), we will reach the distribution function that characterises the surface of the workpiece.

Profilograms that describe the workpiece and the tool should be taken at the beginning of calculations. Elementary functions cannot be used when calculating integrals, therefore approximations and numerical methods should be applied.

STOCHASTIC MODEL OF THE HONING

After discussing the general case, the honing will be discussed, using the above discussed method. The movement of the workpiece compared to the tool (trajectory) may practically be described by a line with β slope angle, thus (fig. 2.):

$$tg\beta=\frac{v_{r1}\sin(\pi-\alpha)}{B_sZ_sn_0}=b, \quad (7)$$

where v_{r1} is the linear radial feeding speed of the tool ($v_{r1}\cong$ constant), α is the half of the section angle of surface etching, B_s is the width of the honing prism, Z_s is the number of honing prisms on the tool, n_0 is the revolution per minute of the spindle.

The "a" distance between the tool and the workpiece is the allowance to be removed (for one side). The macro-geometrical form of the tool in the longitudinal section is a line (just the axe x), thus S(x)=0, and consequently y=g(x). The equation of trajectory: y=W=(x;y)=a-h-bx (the system of axes could be taken for y_0=0). The allowance on one side is: 0,5Z=a.

The density function of the cutting grains' peaks is:

$$f_1(0;v)=f(a-h-bx;\ v-b). \quad (9)$$

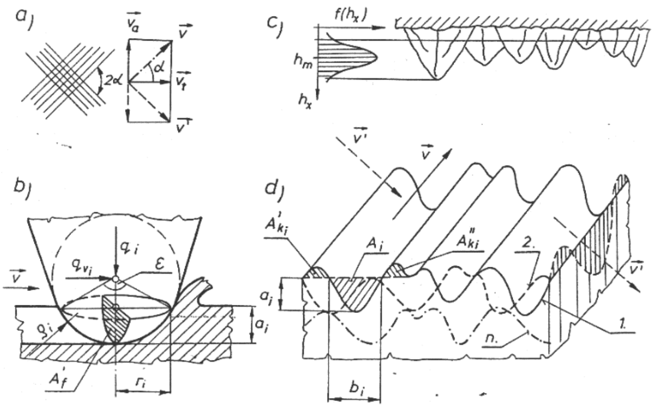

***Figure 2**. Stochastic model and interaction of a multi-edge tool with undetermined edge-geometry and the workpiece ($\vec{v}_c=\vec{v}$ is the cutting speed): a, trajektory; b, model of microcutting; c, heigh distribution of cutting-grains; d, surface roughness of the workpiece.*

The equations of the trajectory and that of the tool:

$$y=W(x;h) \text{ and } y=S(x)+g(x). \quad (8)$$

If the tool meets the workpiece (point M_i) once, then k=n=1. The integration limits by the x axis expressed as a function of the honing run t shall be obtained:

$$P(h)=P_0(h)exp[-\int_{0}^{v_{r1}\frac{t}{b}}\int_{0}^{\infty} vf(a-h-bx;v-b)dvdx], \quad (10)$$

where $x_{min}=0$ and $x_{max}=v_{r1}(t/b)$.

The f(y;v) density-function of the tool should be known for the approximate calculation. Based on experimental data it is supposed that the stochastic component of the abrasive tool can be described by a Gaussian (or normal distribution) function [3]:

$$f(y;v)=\frac{1}{\sqrt{2\pi}\sigma_y}exp\left(-\frac{y^2}{2\sigma_y^2}\right)\cdot\frac{1}{\sqrt{2\pi}\sigma_v}exp\left(-\frac{v^2}{2\sigma_v^2}\right) \quad (11)$$

where σ_y^2 is the variance of the y variable of the tool profile σ_v^2 is the variance of the introduced v variable. Interpretation of variances

$$\sigma_y^2=\sigma_y^2(y)=\frac{1}{n}\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2 \text{ and}$$

$$\sigma_v^2=\sigma_v^2(v)=\frac{1}{n}\sum_{i=1}^{n}\left(v_i-\bar{v}\right)^2,$$

where $\overline{y}$ and $\overline{v}$ are the mean values:

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \text{ and } \overline{v} = \frac{1}{n}\sum_{i=1}^{n} v_i .$$

Substituting the latest equation into the integral of formula (10), the expression in the exponent is the following:

$$\frac{\sigma_v}{2b\sqrt{2\pi}}\left[\Phi\left(\frac{a-h}{\sqrt{2}\sigma_y}\right)-\Phi\left(\frac{a-h-v_{r1}\cdot t}{\sqrt{2}\sigma_y}\right)\right], \qquad (12)$$

where $\Phi = \Phi(z) = \frac{2}{\sqrt{\pi}}\int_0^z exp\left(-x^2\right)dx$ error-integral [3].

At the machined surface the relation between average roughness R_a and smoothness parameter R_q (previously: h_q) is: $R_a = k_a R_q$.

According to Linnik and Huszu $R_a \cong 0{,}8R_q$, where $k_a=0{,}8$, or according to Dyachenko: $R_a = (0{,}9 \div 1)R_q$, then $k_a = (0{,}9 \div 1)$ .Taking also into consideration experimental data, in practice the $k_a \cong 1$ value is acceptable as well, thus the $R_a \cong R_q$ approximation holds good.

This method provides the ability to calculate and design the statistical parameters of the machined surface and the process.

Based on the above discussed, we can also describe the distribution of the h_1 penetration depth of the cutting grains into the workpiece (i.e. the width of the chip):

$$P(h_1) = \int_{-\infty}^{\infty}\left\{P\left(h_1 - y\right)\left[\int_{-\infty}^{\infty} f(y;v)dv\right]\right\}dy . \qquad (13)$$

CONCLUSIONS

It is worth to examine the equation (7) from the technological side. What does this formula explain? Based on experiments and theoretical considerations, the following can be stated.

The radial feeding speed of the tool (μm/min) can be described by the $v_{r1} = v_{r1}(A_i;S_j;T_k)$ relationship, where A_i corresponds to the material parameters; S_j a depends on tool parameters, on the T_k technological factors set by the technologist, on the applied cooling-lubricating liquid, as well as on the rough machining (quality of that).

The most important parameters that effect on the speed of removal of stock can be outlined as the hardness of the material among the material parameters, the material, the average grain size and the tool structure among the tool parameters.

Remaining parameters of the equation (7) and their effects have been discussed before. B_s, Z_s and n_0 are in inverse proportion with b. Increasing of the 2α section angle ($\vec{v}_t$ increases of $\vec{v}_a$ decreases) leads to decrease of the value of b. Consequently the characteristics, causing changes in v_{r1} shall be increased or decreased by the technologist, depending on the technological task.

The aim of the rough honing is to provide a relatively great productivity. In case of a given material of workpiece and hardness the v_{r1} radial feeding speed increases if we enhance:

- » the cutting capability of the grains, applying super-hard grains;
- » the average grain size of the tool;
- » the p tool pressure;
- » the $\vec{v}_a$ axial speed component, etc.

The aim of the final - or fine - honing is to gain a good surface quality, i.e. small roughness. Different measures to the above mentioned ones are necessary to achieve this, as for example:

- » decrease the cutting capability of the grains, applying smaller, average grain size;
- » decrease the p tool pressure;
- » decrease the $\vec{v}_a$ axial speed component,
- » increase the $\vec{v}_t$ tangential speed component and $v_c = (v_t^2 + v_a^2)^{0,5}$ the cutting speed.

Applying traditional grain material, the cutting capability of the grains will also decrease. Certainly smaller roughness can be achieved. In this case the production probability of the smaller $\overline{h}$ and R_a values will increase. The system of mathematically formulated relationships corresponds to the experimental observations.The elaborated method is applicable also to describe the abrasive wear processes at grinding or at machining [4, 5].

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