ACTA TECHNICA CORVINIENSIS – Bulletin of Engineering
Tome VI (Year 2013) – FASCICULE 3 [July–September]
ISSN 2067–3809

[1.] R.N. BARIK

RADIATION EFFECT AND MHD FLOW ON MOVING VERTICAL POROUS PLATE WITH VARIABLE TEMPERATURE

[1.] DEPARTMENT OF MATHEMATICS, TRIDENT ACADEMY OF TECHNOLOGY, INFOCITY, BHUBANESWAR-751024, ODISHA, INDIA

ABSTRACT: An attempt is made to study the combined effects of magnetohydrodynamics (MHD) and thermal radiations on unsteady flow of an electrically conducting fluid past an impulsively started infinite vertical porous plate with variable temperature is investigated. A magnetic field of uniform strength is applied along an axis perpendicular to the plate. The plate temperature is raised linearly with time. An exact solution is obtained by Laplace transformation technique. The dependence of the amplitude and skin-friction on various parameters is discussed in detail with the help of graphs.
KEYWORDS: MHD/thermal radiations/porous plate/ variable temperature

INTRODUCTION

Study of MHD flow with heat and mass transfer play an important role in various industrial applications. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermo nuclear fusion and electromagnetic casting of metals.MHD finds applications in electromagnetic pumps, crystals growing, MHD couples and bearing, plasma jets and chemical synthesis. Radiative heat and mass transfer play an important role in manufacturing industries for the design reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications.

Hydromagnetic incompressible viscous flow has many important engineering applications such as magnetohydrodynamic power generators and the cooling of reactors also its applications to problems in geophysics, astrophysics etc. On the other hand, the study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions dissociating fluids. Since some fluids can emit or absorb thermal radiation, it is of interest to study the effects of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing radiation, Hence, heat transfer by thermal radiation is becoming of greater importance in space applications and higher operating temperatures.

The non-linearity of the Navier-Stokes equations presents mathematical difficulties in obtaining the exact solution. One of the exact solutions of the Navier-Stokes equations of the flow due to an impulsively started infinite flat plate was first studied by Stokes [1]. Georgrantopoulous [2] has discussed the free convection effects of the oscillating flow in the Stokes problem past an infinite vertical porous plate with constant suction. Kafousias et.al [3] has extended the above problem in the presence of a transverse magnetic field without taking into account the induced magnetic field.

A number of researchers have made contributions in solving the problems of free-convection flows under various boundary conditions. Notably among them are Singh [4], Muthucumaraswamy and Kulandaivel [5], Singh and Garg [6]. Very recently Singh and Garg [7] have analyzed the Hall current effects on free convection flow past an accelerated porous plate in a rotating system with Heat source/sink by Laplace transformation technique. The current interests in the study of magnetohydynamic (MHD) of relating fluid has been motivated by several important problems such as maintenance and secular variations of earth's magnetic field, the internal rotation rate of sun, the structure of rotating magnetic stars, the planetary and solar dynamo problems and centrifugal machines etc. On the other hand, in view of the increasing technical applications using magnetohydrodynamic (MHD) effect, it is desirable to extend many of the available hydrodynamic solution to include the effects of magnetic field for those cases where the viscous fluid is electrically conducting. The effect of a transverse magnetic field on free convective flows of an electrically conducting viscous fluid, has been discussed in recent and past years by several authors, notably by Gupta [8], Soundalgekar [9], Mishra and Mudili [10], Mahendra Mohan [11], Sarojamma and Krishna [12] and Singh and Garg [13] and Garg et al [14]. Such type of flows has wide range of applications in aeronautics, fluid fuel nuclear reactors and chemical engineering. The various applications of MHD flows in technological fields have been complied by Moreau [15]. Recently, Ramana Reddy et al. [16] have studied the mass transfer and radiation effects of unsteady MHD free convective fluid flow embedded in porous medium with heat generation/absorption. Radiation effects on MHD free convection flow over a vertical plate with heat and mass flux was studied by Sivaiah et al. [17]. Recently, radiation and mass transfer effects on MHD free convection flow through porous medium past an exponentially accelerated vertical plate with variable temperature has been studied by Pattnaik et al. [18]. Reddy et al. [19] have studied the radiation and chemical reaction effects on MHD heat and mass transfer flow inclined porous heated plate.

The purpose of the present investigation is to study the effects of magnetic field on unsteady free convective moving vertical porous plate in the presence of variable temperature and thermal radiations. The effects of different pertinent physical parameters on the velocity, temperature and skin-friction are presented graphically.

MATHEMATICAL ANALYSIS

Consider an unsteady, free convective flow of an in compressible, electrically conducting, viscous fluid past an impulsively started infinite insulated vertical porous plate with variable temperature. Let us introduce a co-ordinate system with plate lying vertically on x'y' plane such that x-axis is oriented in the direction of buoyancy force and the z-axis is perpendicular to the plane of the plate. A uniform magnetic field $\vec{H}$ is acting transverse to the plate in the presence of thermal radiations. Initially, the plate and the fluid were at rest and temperatures of both are also same. At time t' > 0, the plate is given impulsive motion in the vertical direction against gravitational field with constant velocity u'_0, the plate temperature is raised linearly with time. Since the plate occupying the plane z' = 0 is of infinite extent, all the physical quantities depend only on z' and t'. The equation of continuity $\nabla.\vec{V}=0$ gives on integration $w' = -w_0$, where $\vec{V} \equiv (u', v', w')$. The constant $w_0 > 0$ represents the suction velocity at the plate. Using the relation $\nabla.\vec{H}=0$ for the magnetic field $\vec{H} \equiv (H'_x, H'_y, H'_z)$, we obtain $H'_z = H_0$ everywhere in the fluid (H_0 is a constant). In addition, thermal radiation term is added in the energy equation.

The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesqs' approximations, the governing equations to the problem are obtained as

$$u'_{t'} - w_0 u'_{z'} = g\beta(\theta' - \theta'_\infty) + \nu\, u'_{z'z'} - \frac{\sigma\, \mu_e^2 H_0^2}{\rho} u' \quad (1)$$

$$v'_{t'} - w_0 v'_{z'} = \nu v'_{z'z'} - \frac{\sigma \mu_e^2 H_0^2}{\rho} v' \quad (2)$$

$$\rho c_p(\theta'_{t'} - w_0 \theta'_{z'}) = k\theta'_{z'z'} - q'_{z'}, \quad (3)$$

The initial boundary conditions of the problem are

$$u' = v' = 0, \quad \theta' = \theta'_\infty \quad for\ all\ z'\ at\ t' \le 0,$$

$$u' = u'_0, \quad v' = 0, \quad \theta' = \theta'_\infty + (\theta'_{w'} - \theta'_\infty)\ ct'$$

$$at\ z' = 0\ for\ t' > 0,\ u' = 0,\ v' = 0,\ \theta' = \theta'_\infty$$

$$as\ z' \to \infty\ for\ t' > 0. \quad (4)$$

where g, the acceleration due to gravity; β, the coefficient of volume expansion of the fluid; ν, the kinematic viscosity; ρ, the density; θ' the fluid temperature inside the thermal boundary layer; θ'_∞, fluid temperature away from the porous wall; k, the thermal conductivity, c_p the specific heat of the fluid under constant pressure and $c = \frac{u'^2_0}{\nu}$.

The local radiant for the case of an optically thin gray gas is expressed by

$$q'_{z'} = -4a^*\sigma^*(\theta'^4_\infty - \theta'^4). \quad (5)$$

where a^* is the absorption coefficient and σ^* is the Stefan-Boltzmann constant.

We assume that the temperature differences within the flow are sufficiently small such that θ'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding θ'^4 in a Taylor series about θ'_∞ and neglecting higher order terms, thus

$$\theta'^4 \cong 4\theta'^3_\infty \theta' - 3\theta'^4_\infty. \quad (6)$$

By using equations (5) and (6), equation (3) reduces to

$$\rho c_p(\theta'_{t'} - w_0\theta'_{z'}) = k\ \theta'_{z'z'} + 16a^*\sigma^*\theta'^3_\infty(\theta'_\infty - \theta') \quad (7)$$

On introducing the following non-dimensional quantities

$$\left.\begin{aligned} &u = \frac{u'}{u'_0},\quad t = \frac{t' u'^2_0}{\nu},\quad v = \frac{v'}{u'_0},\\ &z = \frac{z' u'^2_0}{\nu},\quad \theta = \frac{\theta' - \theta'_\infty}{\theta'_w - \theta'_\infty},\quad w = \frac{w_0}{u'_0},\\ &Gr = \frac{\nu\ g\beta(\theta'_w - \theta'_\infty)}{u'^3_0},\quad M = \frac{\sigma\mu_e^2 H_0^2 \nu}{u'^2_0 \rho},\\ &R = \frac{16a^*\nu^2\sigma^*\theta'^3_\infty}{k\ u'^2_0},\quad \Pr = \frac{\rho\nu\ c_p}{k}, \end{aligned}\right\} \quad (8)$$

into equations (1), (2) and (7), we get,

$$u_t - wu_z = Gr\theta + u_{zz} + Mu, \quad (9)$$

$$v_t - wu_z = v_{zz} - Mv, \quad (10)$$

$$Pr(\theta_t - w\theta_z) = \theta_{zz} - R\theta, \quad (11)$$

where Gr = Thermal Grashof number, M = Hartmann number , Pr = Prandtl number,
R = Radiation parameter.

The transformed boundary conditions

$$\left.\begin{aligned} &u = 0, v = 0, \theta = 0, \quad for\ all\ z,\ at\ t \le 0\\ &u = 1, v = 0, \theta = t, \quad at\ z = 0,\ for\ t > 0,\\ &u \to 0,\ v \to 0,\ \theta \to 0 \quad as\ z \to \infty,\ for\ t > 0. \end{aligned}\right\} \quad (12)$$

The equations (9)-(11) subject to the boundary conditions (12) describe the hydromagnetic free-convection flow past the moving porous plate.

Introducing the complex velocity F = u + iv, we find that equations (9) and (10) can be combined into a single equation of the form

$$F_t - wF_z = Gr\,\theta + F_{zz} - MF. \qquad (13)$$

The corresponding transformed boundary conditions in the complex notations are given as

$$F = 0,\ \theta = 0 \ \text{ for all } z,\ \text{at } t \le 0,$$
$$F = 1,\ \theta = t \quad \text{at } z=0,\ \text{for } t>0, \qquad (14)$$
$$F \to 0,\quad \theta \to 0 \quad \text{as } z \to \infty,\ \text{for } t > 0.$$

In order to find the solution of equation (13), we take Pr = 1. This is possible assumption since the Prandtl number is a measure of relative importance of viscosity and heat conductivity in the fluid. For most gases the Prandtl number is of unit order, so that the velocity and the thermal boundary layer will be of the same order of thickness (cf. Houghton and Boswell [20]). By using the Laplace transformation technique solutions of equations (11) and (13) subject to the boundary conditions (14) are derived as:

$$F = A_1 A_3 (\eta_1 \exp X_1 \operatorname{erfc}\eta - \eta_2 \exp X_2 \operatorname{erfc}\eta_2) - A_2 A_3 (\eta_3 \exp X_3 \operatorname{erfc}\eta_3 - \eta_4 \exp X_4 \operatorname{erfc}\eta_4)$$
$$+\frac{1}{2}(\exp X_1 \operatorname{erfc}\eta_1 + \exp X_2 \operatorname{erfc}\eta_2), \qquad (15)$$

$$\theta = A_4\{\eta_5 \exp X_5 \quad ecfc\eta_5 - \eta_6 \exp X_6 \quad erfc\eta_6\} \qquad (16)$$

Using equation (15), we get the following expression for the skin friction components τ_x and τ_y as:

$$\tau_x + i\tau_y = -\left.\frac{dF}{dz}\right|_{z=0} = A_3\left[\begin{array}{l} A_5 \quad erf \quad X_7 + \\ \left(\frac{t}{\pi}\right)^{1/2}\left\{\begin{array}{l}\exp(-X_7^2) \\ -\exp(-X_8^2)\end{array}\right\} - \\ A_6 \quad erf \quad X_8 \end{array}\right]$$
$$+ b_1 erf X_7 + \frac{w}{2} + \frac{1}{\sqrt{\pi t}}\exp(-X_7^2) \qquad (17)$$

where all A_i's, X_i's and η_i's are given in the appendix.

In equations (15)-(17), the argument of the complementary error function and error function is complex. Hence in order to obtain the x- and y-components of velocity, temperature and skin-friction, it is necessary to introduce some properties of the error function with complex arguments due to Abramowitz and Stegun [21]: i.e.

$$erf(c+id) = erf \quad c \quad + \frac{\exp(-c^2)}{2\pi c}\left\{\begin{array}{l}1-\cos(2cd) \\ +i\sin(2cd)\end{array}\right\}$$
$$+\frac{2\exp(-c^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4c^2}\{f_n(c,\ d) + ig_n(c,\ d)\} \qquad (18)$$

where:

$$f_n(c, d) = 2c - 2c\cosh(nd)\cos(2cd) + n\sinh(nd)\sin(2cd), \qquad (19)$$
$$g_n(c, d) = 2c\cosh(nd)\sin(2cd) + n\sinh(nd)\cos(2cd), \qquad (20)$$

and $|\in(c, d)| \cong 10^{-16}\,\mathrm{erf}(c, id)$.

RESULTS AND DISCUSSION

The expressions for velocity, temperature and the skin friction τ are calculated numerically for different values of suction parameter w, Hartmann number M, Grashof number Gr, Radiation parameter R and time t. The resultant velocity profiles are shown graphically in Figure-1. It is observed that the amplitude F of the velocity decreases with the increase of Grashof number Gr and suction parameter w. The velocity F increases with the increase of Hartmann number H and time t. But no alteration in the velocity is found with the increase of Radiation parameter R. The temperature profiles are presented in Figure-2. This figure clearly shows that the temperature decreases with the increase of suction parameter w,

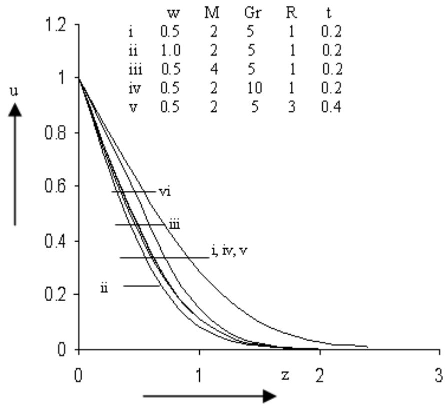

Figure 1. Velocity profiles

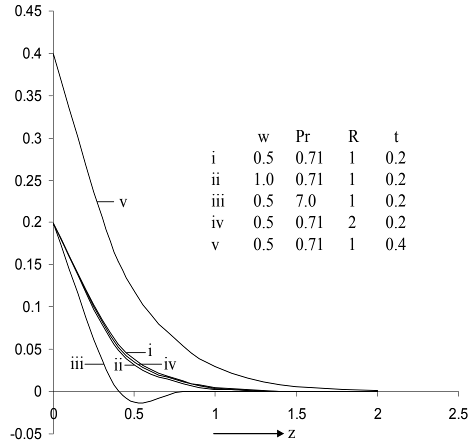

Figure 2. Temperature profiles

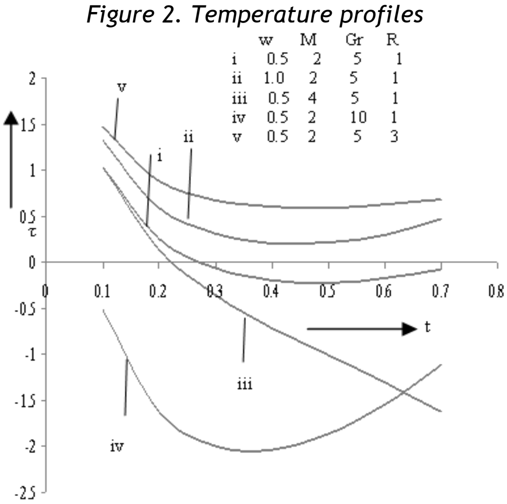

Figure 3. Variations of skin-friction

Prandtl number Pr and Radiation parameter R. But increase with the time t only. Figure-3 shows the variation of the skin-friction with various values of different parameters. This figure clearly depicts that the skin friction τ increases with the increase of suction parameter w,

Radiation parameter R. But τ decreases with the increase of Hartmann number M and Grashof number Gr.

Appendix

$$b_1 = \left(\frac{w^2}{4} - M\right)^{\frac{1}{2}},\ b_2 = \left(\frac{w^2}{4} + R\right)^{\frac{1}{2}},\ b_3 = \left(R + \frac{w^2 Pr^2}{4}\right)^{\frac{1}{2}}$$

$$\eta_1,\eta_2 = \frac{z \pm 2b_1 t}{2t^{\frac{1}{2}}},\ \eta_3,\eta_4 = \frac{z \pm 2b_2 t}{2t^{\frac{1}{2}}}, \eta_5,\eta_6 = \frac{z\,Pr \pm 2b_3 t}{2t^{\frac{1}{2}}\,Pr^{\frac{1}{2}}}$$

$$A_1 = \frac{t^{\frac{1}{2}}}{2b_1},\quad A_2 = \frac{t^{\frac{1}{2}}}{2b_2},\quad A_3 = -\frac{Gr}{M+R},\quad A_4 = \frac{t^{\frac{1}{2}}}{2b_3},$$

$$A_5 = tb^{\frac{1}{2}} + \frac{1}{2b_1},\ A_6 = tb_2 + \frac{1}{2b_2},$$

$$X_1 = \left(b_1 - \frac{w}{2}\right)z,\ X_2 = -\left(b_1 + \frac{w}{2}\right)z,\ X_3 = \left(b_2 - \frac{w}{2}\right)z,$$

$$X_4 = -\left(b_2 + \frac{w}{2}\right)z,\ X_5 = \left(b_3 - \frac{w\,Pr}{2}\right)z,$$

$$X_6 = -\left(b_3 + \frac{w\,Pr}{2}\right)z,\ X_7 = b_1 t^{\frac{1}{2}}, X_8 = b_2 t^{\frac{1}{2}}$$

CONCLUSIONS

(i) The amplitude of the velocity field decreases with the increase of Grashof number and suction parameter but the reverse effect is observed in case of Hartmann number and time span.

(ii) The radiation parameter has no significant effect on velocity field.

(iii) The temperature field decreases with the increase of either suction parameter or Prandtl number or radiation parameter but the reverse effect is observed in case of time span.

(iv) The skin friction co-efficient increases with the increase of either suction parameter or radiation parameter whereas the opposite effect is marked in case of Hartmann number and Grashof number.

REFERENCES

[1.] Stokes GG. On the effect of the internal friction of fluids on the motion of pendulums, Camb. Phil. Trans. 1851; ix: 8.

[2.] Georgantopoulous, G.A. Effects of free-convection on the hydromagnetic accelerated flow past a vertical porous limiting surface. Astrophysics and Space Science 1979; 65: 433-441.

[3.] Kafousias NG, Massalas CV, Raptis AA, Tzivanidis GL, Georgantopoulous GA & Goudas GL. Free convection effects on the hydromagnetic oscillatory flow in the Stokes problrem past an infinite porous vertical limiting surface with constant suction-1. Astrophys. Space Sci 1980; 86: 99-110.

[4.] Singh Ajay Kumar. Hydromagnetic mixed monvection flow and heat transfer with periodic suction velocity, permeability and heat sink. J Energy Heat and Mass Transfer, 2008; 30: 21-43.

[5.] Muthucumaraswamy R & Kulandaivel T. Radiation effects on moving vertical plate with variable temperature and uniform mass diffusion. J Energy Heat and Mass Transfer, 2008; 30: 79-88.

[6.] Singh KD & Garg BP. Exact solution of an oscillatory free-convective MHD flow in a rotating porous channel with radiative heat. Proc Nat Acad Sci 2010; 80 A: 81-89.

[7.] Singh KD & Garg BP. Effects of Hall current on free-convective flow past an accelerated vertical porous plate in a rotating system with heat source/sink, J Raj Acad Phy Sci 2009; 8(2): 191-202.

[8.] Gupta AS. Combined free and forced convection effects on the magnetohydrodynamic Flow through a Channel". ZAMP 1969; 20: 506-513.

[9.] Soundalgekar VM. Free convection effects on steady MHD flow past a vertical porous plate. J. Fluid Mechanics 1974; 66: 541-551.

[10.] Mishra SP & Mudili JC. Combined free and forced convection effects on the magneto-hydrodynamic flow through a porous channel. Proc. Ind. Acad. Sci, 1976; 84-A: 257-272.

[11.] Mahendra Mohan. Combined effects of free and forced convection on magnetohydrodynamic flow in a rotating channel. Proc. Indian Acad. Sci. 1977; 84: 383-401.

[12.] Sarojamma G & Krishna DV. Transient hydromagnetic convective flow in a rotating channel with porous boundaries. Acta Mech. 1981; 40: 277-288.

[13.] Singh KD & Garg BP. Radiation effects on unsteady MHD free convective flow through porous medium past a vertical porous plate. Proc. Indian Natn. Sci. Acad. 2009; 75(1): 41-48.

[14.] Garg BP, Singh KD & Pathak Reena. An analysis of radiative, free-convective and mass transfer flow past an accelerated vertical plate in the presence of transverse magnetic field. J. Rajasthan Acad. Phy. Sci. 2011; 10(1): 1-10.

[15.] Moreau R. Magnetohydrodynamics. Kluwer Academic Publishers, Dordrecht, 1990.

[16.] Ramana Reddy, G.V., Ramana Murthy, Ch. V. and Bhaskar Reddy, N., Mass transfer and radiation effects of unsteady MHD free convective fluid flow embedded in porous medium with heat generation/absorption, Journal of Applied Mathematics and Fluid Mechanics, vol.2, No.1, pp. 85—98 (2010).

[17.] Sivaiah, M., Nagarajan, A.S. and Reddy, P.S., Radiation effects on MHD free convection flow over a vertical plate with heat and mass flux, Emirates Journal for Engg. Research, vol.15 (1), pp. 35-40 (2010).

[18.] Pattnaik, J.R., Dash, G.C. and Singh, S., Radiation and mass transfer effects on MHD free convection flow through porous medium past an exponentially accelerated vertical plate with variable temperature, Annals of Faculty Engineering Hunedoara - International Journal Engineering, Tome X. Fascicule 3. ISSN 1584-2673. Pp. 175-182 (2012).

[19.] Reddy, S., Reddy, G.V.R. and Reddy, K.J., the radiation and chemical reaction effects on MHD heat and mass transfer flow inclined porous heated plate, Asian Journal of Current Engineering and Maths 1:3, pp. 115-119 (2012).

[20.] Houghton EL & Boswell RP. Further aerodynamics for engineering students, Edward Arnold Ltd. 1969; 252.

[21.] Abramowitz BM & Stegun IA. Handbook of mathematical functions with formulas, graphs and mathematical tables. U.S. Govt. Printing Office, Washington D.C., USA. 1964.