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[1.] István BÍRÓ

SIMPLE NUMERICAL METHOD FOR 2-DOF PLANAR KINETICAL PROBLEMS

[1.] TECHNICAL INSTITUTE, FACULTY OF ENGINEERING, UNIVERSITY OF SZEGED, H-6724 SZEGED, MARS SQ. 7, HUNGARY

ABSTRACT: The motion analysis is an important field in the education of mechanical engineers. Kinematical and kinetical analysis of rigid bodies linked to each other by different constraints (mechanical systems) leads to integration of second order differential equations. In this way the kinematical functions of parts of mechanical systems can be determined. The degrees of freedom of the mechanical system increase as a result of the application of elastic parts in it. Numerical methods can be applied to solve such problems. A simple numerical method will be demonstrated by author by the aid of several examples. Some parts of results obtained by using the numerical method were checked by analytical way. The published method can be used in the technical higher education.
KEYWORDS: rigid bodies, motion analysis, kinematical functions

INTRODUCTION

The knowledge of motion analysis plays an important role in the education of mechanical engineers. Dynamical analysis of mechanical systems is a fundamental chapter in motion analysis. Mechanical systems having one or two degrees of freedom can be described by second order differential equations or differential equation systems. Analytical solution of them in most cases is quite difficult or impossible.

In such cases the application of numerical methods is advantage. The results obtained in this way can be demonstrated in different kinematical diagrams. It helps engineer students better learning of school-work and connections among different physical quantities. In this paper the results of kinetical analysis of two mechanical systems will be demonstrated.

THE SIMPLE NUMERICAL METHOD

Let us suppose that the mechanical system can be described by $\mathbf{q}=q(r,\varphi)$ *generalized coordinates. Physical quantities* $\dot{r}_o, r_o, \dot{\varphi}_o, \varphi_o$ *describe the initial state of the system. Time step:* $t_{i+1}-t_i$. *Applied algorithms can be seen in table below.*

Table 1. Applied algorithms

t	$\ddot{r}(\dot{r},r,\dot{\varphi},\varphi)$	$\dot{r}$	r
t_o	$\ddot{r}_o(\dot{r}_o,r_o,\dot{\varphi}_o,\varphi_o)$	$\dot{r}_o$	r_o
t_1	$\ddot{r}_1(\dot{r}_1,r_1,\dot{\varphi}_1,\varphi_1)$	$\dot{r}_1=\dot{r}_o+\ddot{r}_o(t_1-t_o)$	$r_1=r_o+\dot{r}_o(t_1-t_o)$
t_2	$\ddot{r}_2(\dot{r}_2,r_2,\dot{\varphi}_2,\varphi_2)$	$\dot{r}_2=\dot{r}_1+\ddot{r}_1(t_2-t_1)$	$r_2=r_1+\dot{r}_1(t_2-t_1)$
t_3	....	....	....
t_4	....	....	....

Table 2. Applied algorithms

t	$\ddot{\varphi}(\dot{r},r,\dot{\varphi},\varphi)$	$\dot{\varphi}$	φ
t_o	$\ddot{\varphi}_o(\dot{r}_o,r_o,\dot{\varphi}_o,\varphi_o)$	$\dot{\varphi}_o$	φ_o
t_1	$\ddot{\varphi}_1(\dot{r}_1,r_1,\dot{\varphi}_1,\varphi_1)$	$\dot{\varphi}_1=\dot{\varphi}_o+\ddot{\varphi}_o(t_1-t_o)$	$\varphi_1=\varphi_o+\dot{\varphi}_o(t_1-t_o)$
t_2	$\ddot{\varphi}_2(\dot{r}_2,r_2,\dot{\varphi}_2,\varphi_2)$	$\dot{\varphi}_2=\dot{\varphi}_1+\ddot{\varphi}_1(t_2-t_1)$	$\varphi_2=\varphi_1+\dot{\varphi}_1(t_2-t_1)$
t_3	....	....	....
t_4	....	....	....

EXAMPLE 1: SPRING PENDULUM (DOF: 2)

In Fig. 1 the sketch of spring pendulum can be seen.

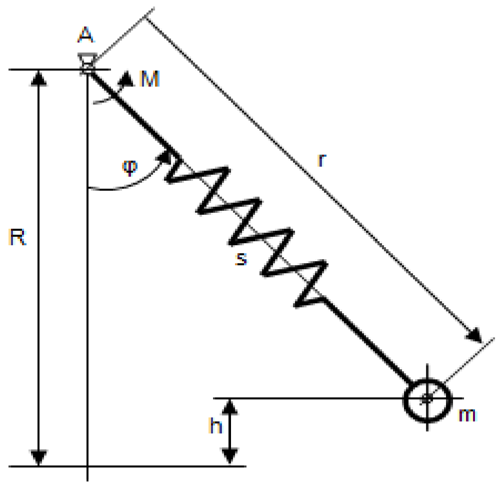


Figure 1. Sketch of spring pendulum

Data: M = -1 Nm, s = 800 N/m, m = 2 kg, R = 0,2 m (length of unloaded rod), g = 10 m/s^2, $\dot{r}_o=0m/s$, $r=0{,}28m$, $\dot{\varphi}_o=0rad/s$, $\varphi_o=1rad$

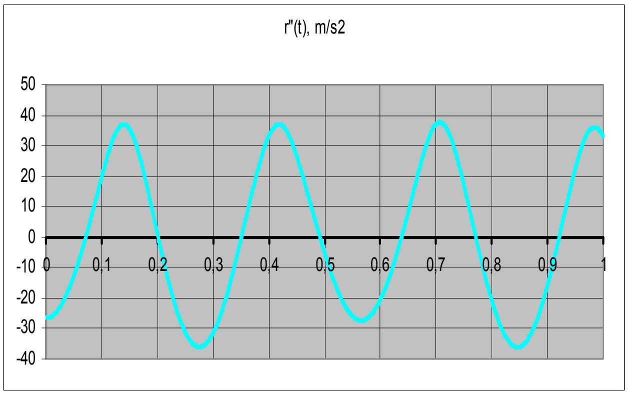
(time step: 0.0001 s, time interval: 0 ≤ t ≤ 1 s)

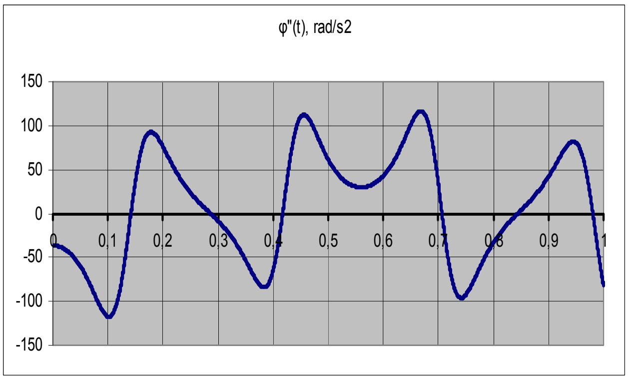
The motion of the spring pendulum is described by the following second order differential equation system. They can be obtained by the aid of second order Lagrangian-equation.

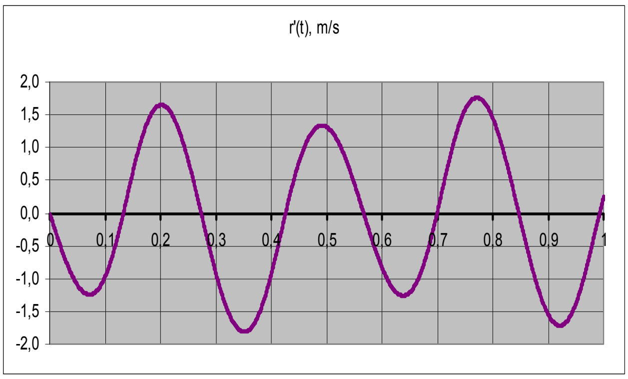
$$\ddot{r} = \frac{mr\dot{\varphi}^2 - s(r - r_o) + mg\cos\varphi}{m},$$

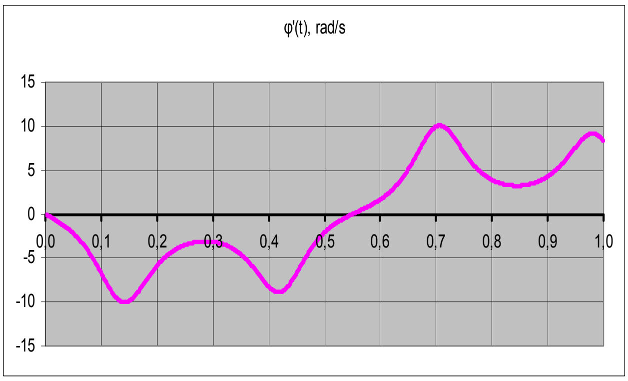
$$\ddot{\varphi} = \frac{M - mgr\sin\varphi - 2mr\dot{r}\dot{\varphi}}{mr^2}$$

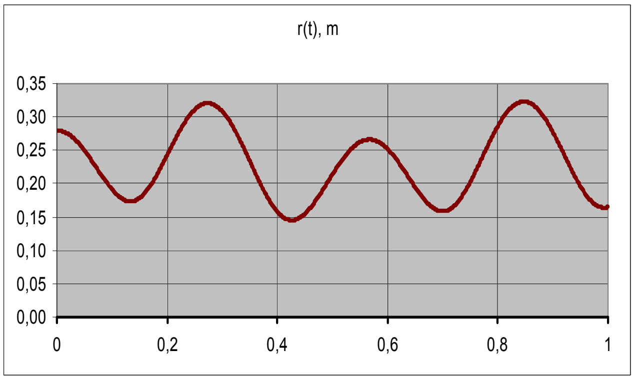
After two-time numerical integration of motion equations the following kinematical functions can be obtained.











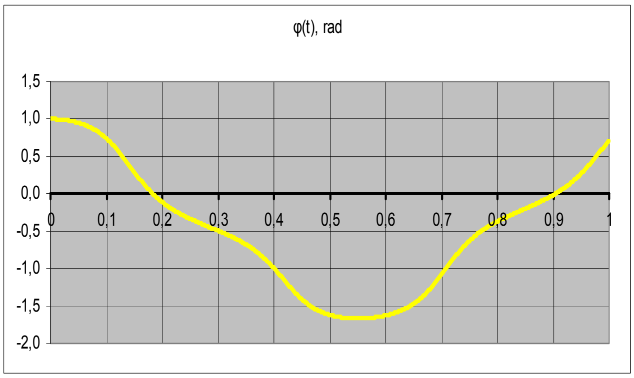


Figure 2. The kinematical functions for Example 1: Spring pendulum

EXAMPLE 2: LINK MECHANISM WITH RECIPROCATING MOTION MASS (DOF: 2)

In Fig. 2 a sketch of a simple mechanical system (two degrees of freedom) can be seen. There is a mass and a link mechanism in between a spring.

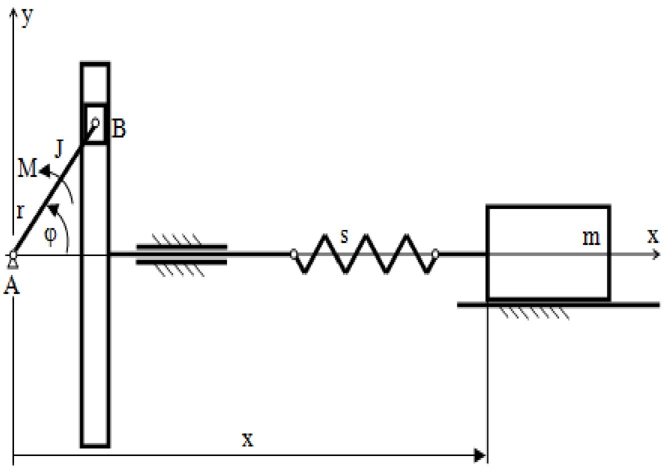


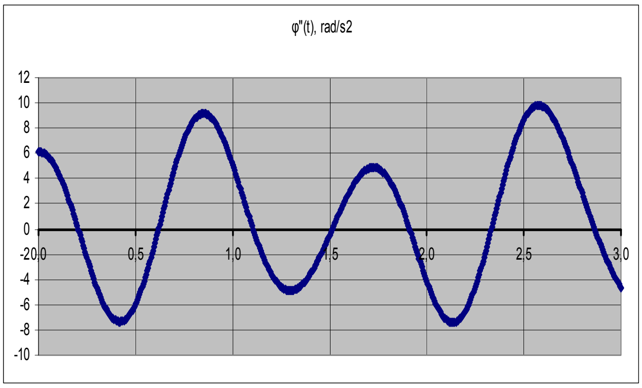
Figure 3. Sketch of link mechanism

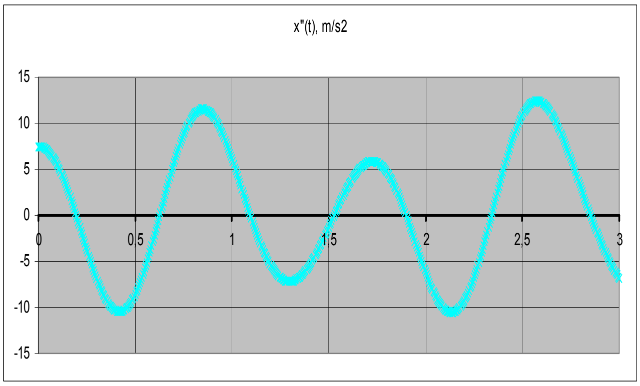
Data: M = 2 Nm, J = 4 kgm^2, r = 0.15 m, s = 1000 N/m, m = 20 kg, $\dot{x}_o = 0m/s$, $x_o = 0m$, $\dot{\varphi}_o = 3rad/s$, $\varphi_o = 0rad$, (time step: 0.005 s, time interval: 0 ≤ t ≤ 3 s). The motion of the link mechanism can be described by the following second order differential equation system.

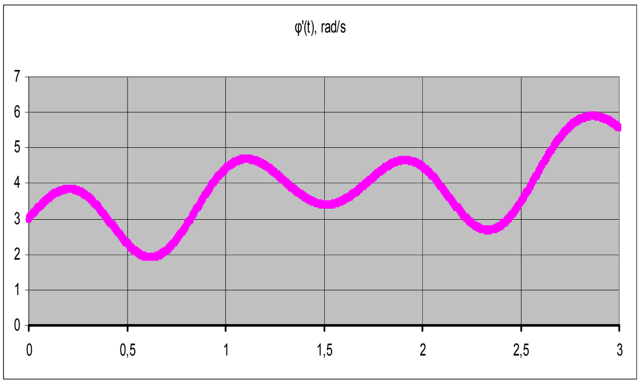
$$\ddot{r} = \frac{sr\cos\varphi - k\dot{x} - sx}{m},$$

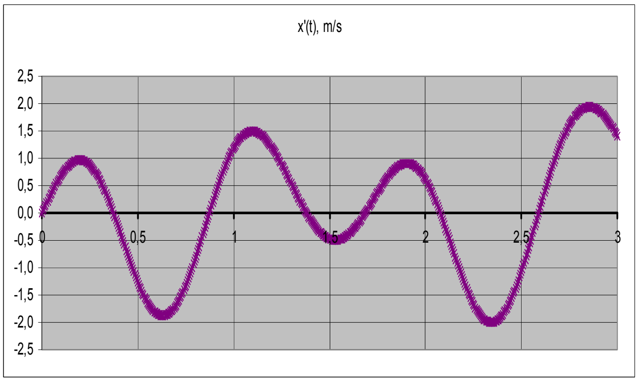
$$\ddot{\varphi} = \frac{M - sr(x - r\cos\varphi)}{J}$$

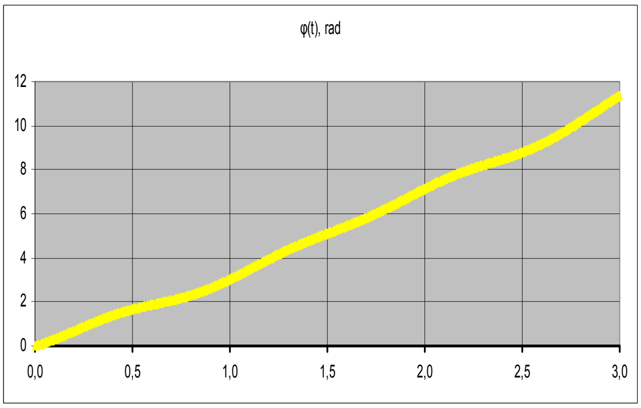
After two-time numerical integration of motion equations the kinematical functions will be the followings.











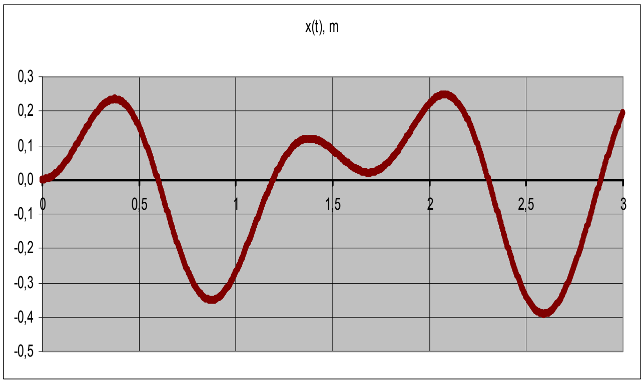


Figure 4. The kinematical functions for Example 2: Link mechanism with reciprocating motion mass

CONCLUSIONS

The above demonstrated method can be applied easily for engineer students in the higher education. The method is suitable for investigation of similar mechanical systems having one or more degrees of freedom. By consequent modification of data (physical quantities) of systems a wide range of possible structures and their kinematical behavior can be analyzed. For this reason the application of this method can be advantageous for engineer students.

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