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MATHEMATICAL OPTIMIZATION IN DESIGN – OVERVIEW AND APPLICATION

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ABSTRACT: *Today, optimization is one of the main tasks in design work to develop functional and economical satisfying products. The paper demonstrates an overview of mathematical based optimization methods and views their applications in the field of design work.*
KEYWORDS: *optimization, mathematics, methods, applications*

INTRODUCTION

The substantial task of the engineers is to solve technical problems considering several material types, technological, economical, legal, conditions combining with ecological and human-related restrictions. The solutions have to fulfill all the given and formulated requirements, otherwise it has to reach an optimum value when the designer applying the methodology of the structural design [1].

The necessary resources during the lifetime of the product, like the material and energy consumption, and also the product development related costs and time are important factors we have to reduce, and have to find the optimal solution while keeping the demanded quality level of the product. Changing the variables of such an optimizing problem the model will result a reduction of the mass or the manufacturing cost or the shape or the material properties of the product and in the end of the process we can reach the minimum value of the optimization task.

OPTIMIZATION PROBLEM AND THEIR DESCRIPTION

The mathematical description of an optimization problem requires us to apply an appropriate model which has limited number of parameters (design variables). These variables have to be relevant to describe the most important characteristics of the design problem.

MATHEMATICAL DESCRIPTION

Any kind of optimization problem can be formulated to find the appropriate set of the design variables in the multidimensional parameter space, which can optimize the main objective function. Generally the minimum (or maximum) of the ojective function $f(\mathbf{x})$ in an n-dimensional, Euclidean space R^n is to be searched. The result of the optimization can be illustrated thus by one point or a vector in this solution space. In the mathematical notation the optimization problem can generally be represented, as:

$$\min f(\mathbf{x}),\ \mathbf{x} \in R^n;$$
$$0 \le g_j(\mathbf{x}),\ j=1,2,\ldots,m;$$
$$0 = h_j(\mathbf{x}),\ j=m+1,\ldots,p;$$

where $\mathbf{x}=[x_1,x_2,\ldots,x_n]^T$ the vector of the unknown quantities, $g_j(\mathbf{x})$ and $h_j(\mathbf{x})$ the restriction constraints, which can be represented mathematically as equations and/or inequations, m and p are integer numbers.

Frequently the objective function in the optimizing problem is the pure mass of the product, or it is a cost (material and production cost), some special cases it is the stiffness of the part [2], or the number of tool changes in the production process [3].

The optimization variables can be geometrical dimensions of the construction [4], when the geometry and the material of the part is fixed [5]. The restrictions depend on the different tasks and can be e.g. frames (building) space, firmness, deformation, stabilities and different kind of manufacturing restrictions.

LOCAL AND GLOBAL MAXIMUM AND MINIMUM

The majority of the optimizing procedures usually supply local optimum solution, but it is possible, that these local optimum points are also global ones.

In the case of a local optimum is reached, the solution point will be a better starting point (start vector) for further calculation in order to reach the global optimum solution.

In the case when we are interesting to find the global optimum solution, than it is a possible strategy to use different starting vectors, and to perform the optimization process several times. However we have to remark, that a global optimum solution will exist only in a convex space of the design variables.

THE ROLE OF THE RESTRICTIONS

The optimum solution can be found quite easily (by differentiating the objective function) when the optimization problem has no restrictions.

Other cases, when the variable space is limited by different restrictions, it is more difficult to find the location of the optimum solution. The linear optimization problem with restrictions seems an important special case of the optimization tasks. This case both the main function and all the restrictions are linear functions of the design variables [6].

The restrictions formulated with equations are handled more simply, than the restrictions expressed by inequations. In the majority of the optimization tasks the possible range of the design variables is limited by one or more restriction functions. For the solution of optimization problems with restriction equations the Lagrangian multiplicator method was used [7]. Also a lot of algorithms were prepared for the optimization problems with inequation restrictions.

OPTIMIZATION PROCEDURE

The major cases of technical problems the objective function as well as the restriction functions are nonlinear functions of the design variables, so the optimization problem can be handled by the methods of nonlinear optimization. One of the most well-known nonlinear optimization methods is the sequential unconstrained minimization technique [8].

The essence of this method is transferring the problem with restrictions into a problem without restrictions. A designer often meets optimization tasks with several objective functions during the product optimization process. The optimization task with several target functions represents an aggravation of the optimization problem. Because of the various application possibilities an intensive development can be observed last years.

In the case of a large number of optimization problems the variables may take only discrete values. This kind of problems is called discrete optimization problems. This discrete optimization procedure will calculate the optimal value of a main function when the design variables can be selected from a discrete, so called material variable range [9].

Recent years the evolutionary algorithms are used frequently as optimization procedures in the case of component and product optimization (mechanical components) [10].

Evolutionary algorithms are stochastic search methods, which are based on the principles of the biological evolution. Three optimization directions of the evolutionary algorithms were developed independently from each other: the evolutionary programming, the evolution strategies and the genetic algorithms. All these methods use the variation and selection operations as the basic elements of the evolution process, but they differ in the development of these elements [11]. The usage of these algorithms will increase in the coming years due to the various application possibilities. The calculation of the actual restriction values (e.g. shifts, tensions, etc.) can be computed in many applications only by numeric methods.

For example in [12] there are some optimization samples of a wrench, flange, etc. on the basis of a FEM-Models. The optimization with the help of the finite element method is a widely used technique [13].

TECHNICAL OPTIMIZATION PROBLEMS

A lot of optimization problem will combine technical and economical requirements against the product or the component, so functional and economical requirements must be equally considered. When specifying the technical and economical approach, product and process optimization is defined. The product optimization can be specified further:

- *product optimization,*
- *topology optimization,*
- *form optimization,*
- *dimension optimization,*
- *material optimization and*
- *process optimization.*

During the topology optimization the arrangement of geometrical elements of a product can be determined (dimensions and position) with the optimizing procedure. The topology optimization is an everyday task of technical designers. He has to design a component (as a part of a product) so that available space mustn't exceed, it has to keep the outside loads, and the minimum material expenditure can be achieved at the same time [14]. For the topology optimization task a typical sample is the optimizations of a trust units (Figure 1).

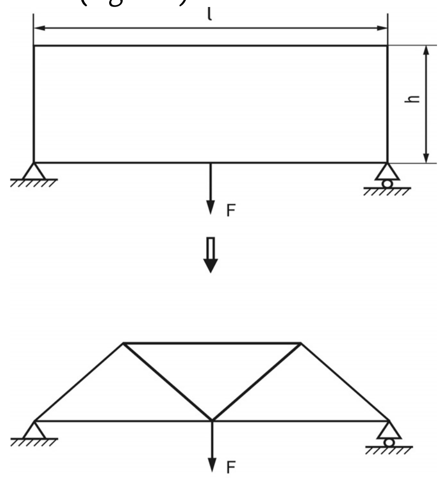

Figure 1. Topology optimization in the case of optimization a trust unit

During the beam unit optimization the topology optimization is an effective design tool, because it is possible to develop light and at the same time rigid beam units [15]. The consumption of time necessary for the development can be clearly reduced [16]. As an initial geometry, the results of the topology *optimization can be usually used for the dimension and form optimization process. A goal of the form optimization is to determine the optimal geometry of a component - under given boundary conditions - regarding defined quality criteria (Figure 2).*

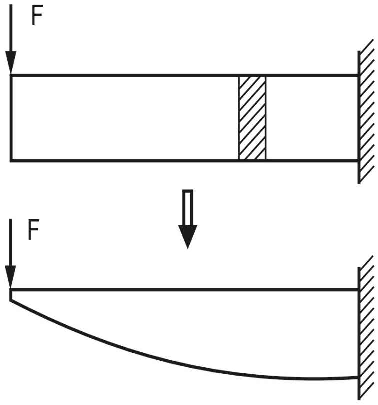

Figure 2. Form optimization of a beam

Objective function and restrictions are selected according the nature of the tasks [17]. Restrictions such as displacement and tensions generally cannot be computed analytically, but it is possible generate them numerically e.g. with the method of the finite elements. For the solution of form optimization problems also variation principles can be used in special cases.

In the case of dimension optimization often the dimensions of cross sections are computed [18]. In the cross-sectional optimization problems mainly displacements, tensions or natural frequencies are determined (Figure 3). An example of this class of optimization problems is the calculation of optimal dimensions of mechanical components or the optimization of the isolation thickness of pipelines.

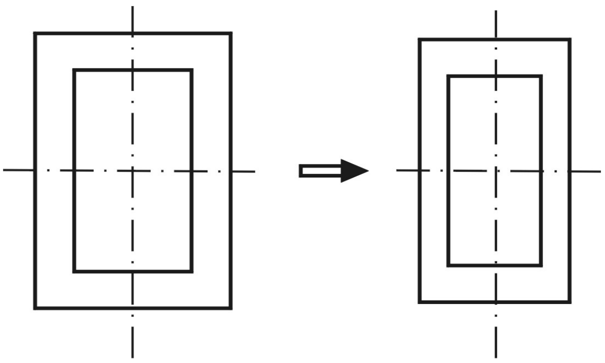

Figure 3. Dimensional optimization sample of a rectangular section

During the material optimization have to find the optimal structure of the materials [19] e.g. how to arrange the structure of composite layers (Figure 4) or how to arrange the fiber strips in fiber-reinforced materials.

The material optimization basically is a topology optimization, however in the variable space is a microscopic solution area.

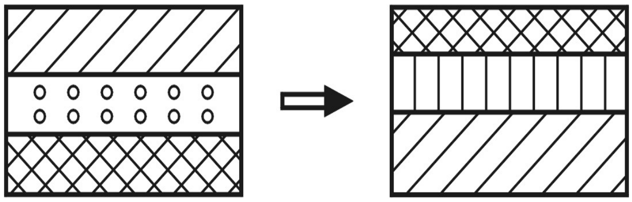

Figure 4. Material optimization sample of a composite structure

The process optimization was developed in technology and economics in the last decades very rapidly, and the theory was based on the discipline „Operation's Research“. The technical processes in the production were completed with the disassembly or the recycling, the technological processes with the equipment technology (Figure 5) or economic processes in the economic science models.

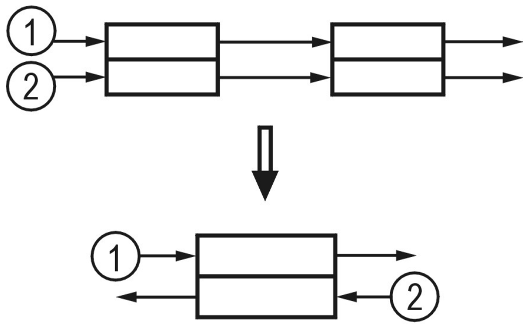

Figure 5. Process optimization in the heat transfer technology

OPTIMIZATION OF A TRUST STRUCTURE

We can find several samples for optimization for trust structures in the literature. Most of the cases two major problem type exists:

- *the topology of the structure is fixed, so the cross sections of the beams are the unknown quantities,*
- *the overall topology of the structure is variable, so we have to find also the optimal overall geometry.*

The calculation of a fixed topology trust structure is presented. (Figure 6) The method of solution is based on [20].

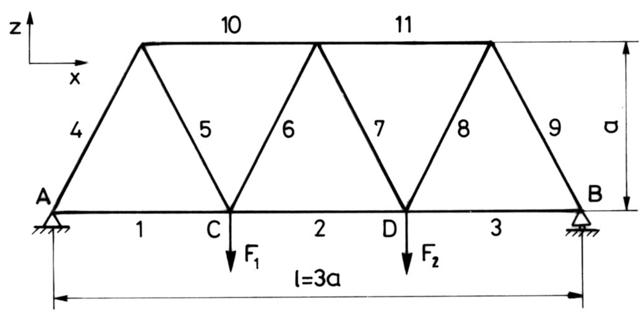

Figure 6. Topology of a trust structure
Unknown quantities: the size of the square shaped cross section beams (11 datas in this case).

Objective function: the total mass of the trust structure and the displacements of the C and D points will be selected as main functions (multiobjective optimization). The displacements of the nodes were calculated with weight factors.

Restrictions: the maximum limits of the nodes C and D will be set. $w_{Cmax} \leq w_{allow}$; $w_{Dmax} \leq w_{allow}$. The stresses of the independent rods of s trust structure is tension or compression. The maximum tension stresses in all of the 11 rods must not exceed the allowable value. $\sigma_{imax} \leq \sigma_{allow}$. In the case of the compressed rods another possible failure, the buckling exists, so the forces (N_i) in the individual rods must be smaller than the limit value ($N_{ib,Rd}$) calculated under the EUROCODE 3 standard, so, $N_i \leq N_{ib,Rd}$.

Table 1. The results of the optimization of a trust beam

Weight factors	Optimal cross sections [mm²]	Mass [kg]	Displacement of node C [mm]	Displacement of node D [mm]
w_1 = 0,995 w_2 = 0,0025 w_3 = 0,0025	A_1 = 98; A_2 = 180; A_3 = 83; A_4 = 1001; A_5 = 218; A_6 = 34; A_7 = 373; A_8 = 185; A_9 = 915; A_{10} = 851; A_{11} = 772.	59,3	4,56	3,24
w_1 = 0,0025 w_2 = 0,5975 w_3 = 0,4	A_1 = 1110; A_2 = 2083; A_3 = 968; A_4 = 2509; A_5 = 2553; A_6 = 354; A_7 = 374; A_8 = 2141; A_9 = 2169; A_{10} = 2249; A_{11} = 1943.	231,3	0,66	0,44
w_1 = 0,0025 w_2 = 0,995 w_3 = 0,0025	A_1 = 1254; A_2 = 2090; A_3 = 813; A_4 = 2801; A_5 = 2775; A_6 = 774; A_7 = 771; A_8 = 1832; A_9 = 1866; A_{10} = 2506; A_{11} = 1622.	240,0	0,60	0,54

The multiobjective optimization task was solved by a so called weight method, where the different objective functions were multiplied by such weight coefficients where the sum of the weight factor is 1. The nonlinear optimization problem was solved for the case of: a=1,5 m, F_1 = 45 kN, and F_2=27 kN. The results are collected in the Table 1, based on [21]. It is clear, how the optimum is moving when the weight factors were changed. While the w_1 factor approaching 1, the role of the mass-related objective is increasing. This case the optimizing process will be a simple mass-minimum optimizing task. The result will be a minimum mass truss structure with relatively big node displacement values. If we will increase the weight factors of the C and D node's displacement, the importance of decreasing these displacements will bigger and of course the optimization process will result a bigger mass, more rigid structure (the second row of the table). If we are intending to reduce the displacement one of the nodes, the corresponding weight factor has to be increased. It can be observed in the third row of the table, that increasing the w_2 weight factor, the displacement of node C will reduce, and the total mass as well as the displacement of node D will increase.

OPTIMIZATION OF THE INSULATION LAYER'S THICKNESS OF A PIPELINE

Calculation of the optimal thickness of an insulation layer has a quite high importance because of the valuable material cost in the investment phase as well as the heat loss cost in the operation phase of a heat pipe system. General case widely used the one-layer insulation, but special cases when high temperature fluid is transported, it is more beneficial to use two-layer insulation system. The insulating materials resisting against the high temperature are relatively expensive ones, but others for lower temperature are cheaper.

The inner insulating material was cork layer; the outer material was polyurethane foam. The PUR material can be applicable up to 130 °C. We cannot find so many papers optimizing the two layer insulating systems in the literature, because of the problem can be handled only with the nonlinear methods. The main goal of this optimization task to find the minimum of an objective function contain the cost of the heat loss and the insulation investment cost. When calculating the insulation task, we have to know the temperature distribution. Frequently only numerical methods are suitable to calculate this temperature field.

It is presented an optimum calculation for a two-layer's insulation system. (Figure 7) The task is to determine the optimal thicknesses (h_1, h_2) of the insulation layer's, while the target cost function – consisting the material costs and the heat loss costs – must be minimum [22].

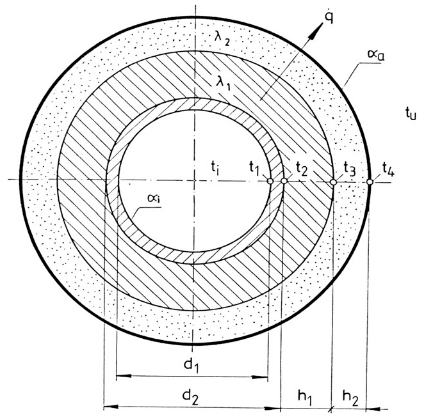

Figure 7. Cross section of two layer insulated heat pipe Unknown quantities: the thicknesses of the inside and outside insulation layers (h_1, h_2).

Objective function: contains the material costs of the pipe, the material costs of the two insulating layers, and the cost of heat loss.

Restrictions: it is necessary to limit the allowable heat loss from different point of view.

($q \leq q_{allow}$). The contact temperature between the two insulation layers (t_3) have to be restricted, because of the relatively low heat resistance of the PUR material (t_{allow}=130 ℃

$t_3 \leq t_{allow}$. The t_3 temperature can be calculated from the fact, that the same heat quantity is transferred for all the layers. Also it is necessary to restrict the temperature of the outside surface (t_4), it has to be bigger, than the outside ambient temperature (t_u). So $t_u \leq t_4$.

The presented sample task calculates a pipe transporting 5 bar pressure steam. The calculation was performed on a 10 m long straight session of the pipe system. The allowable heat loss was q_{allow} = 75 W/m. (Figure 8) The results for the optimum thicknesses (h_{1opt}, h_{2opt}) is presented as a function of the temperature of the steam flow. We can see, that the optimal thickness of the inner insulating layer is independent from the steam (t_i) and outside (t_u) temperature, we have only one optimum value h_{1opt} for that. The reason of this behavior that the other restriction, for the temperature between the two insulating layer is the active restriction. As for the optimal thickness of the outer insulating layer (h_{2opt}) will changing with the different inner/outer temperatures.

This calculation resulting, that it is possible to determine the optimal sizes of the insulation with the correctly formulated objective function and the appropriate restriction conditions [22]. This way the reduction of the material and heat loss costs is possible. This model is also suitable for the calculation of the one layer insulation task, and also for the calculation of a spherical tank with the appropriate modification of the equations [23].

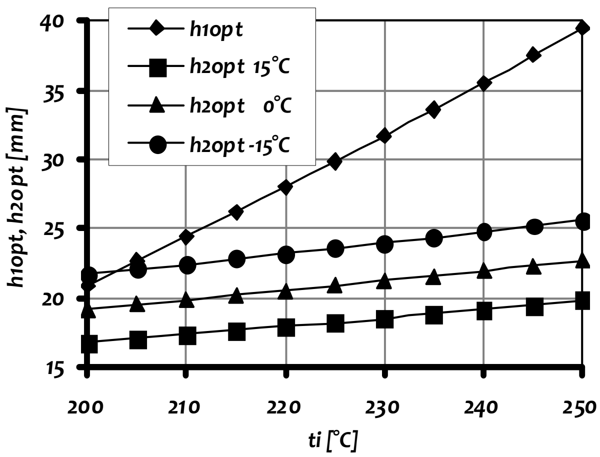

Figure 8. The optimum thickness for the insulating layers vs. the steam temperature

CONCLUSIONS

Mathematical optimization methods are generally applicable in the case of technical and economic problems. The optimization problem in general case is to build up a suitable model: to set up the target function(s) and to formulate the restrictions as mathematical functions or conditions. With the adequate formulation of the optimization problem the functional and economic characteristics of the products and/or the components as well as the processes can be improved fast and effectively.

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